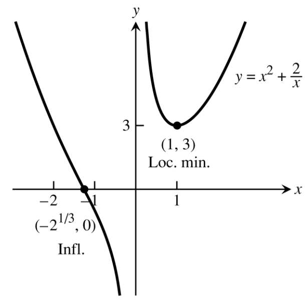
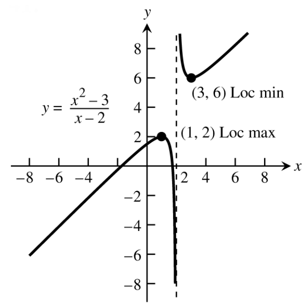
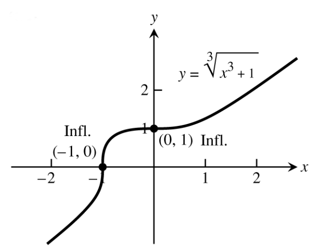
40. When $y = x^2 + \frac{2}{x}$, then $y' = 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2}$ and $y'' = 2 + \frac{4}{x^3} = \frac{2x^3 + 4}{x^3}$. The curve is falling on $(-\infty, 0)$ and $(0, 1)$, and rising on $(1, \infty)$. There is a local minimum at $x = 1$. There are no absolute maxima or absolute minima. The curve is concave up on $\left(-\infty, -\sqrt[3]{2}\right)$ and $(0, \infty)$, and concave down on $\left(-\sqrt[3]{2}, 0\right)$. There is a point of inflection at $x = -\sqrt[3]{2}$.



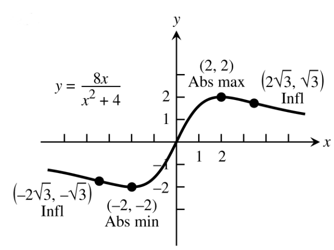
41. When $y = \frac{x^2 - 3}{x - 2}$, then $y' = \frac{2x(x-2) - (x^2 - 3)(1)}{(x-2)^2}$
$= \frac{(x-3)(x-1)}{(x-2)^2}$ and
$y'' = \frac{(2x-4)(x-2)^2 - (x^2 - 4x + 3)2(x-2)}{(x-2)^4} = \frac{2}{(x-2)^3}$.
The curve is rising on $(-\infty, 1)$ and $(3, \infty)$, and falling on $(1, 2)$ and $(2, 3)$. There is a local maximum at $x = 1$ and a local minimum at $x = 3$. The curve is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$. There are no points of inflection because $x = 2$ is not in the domain.



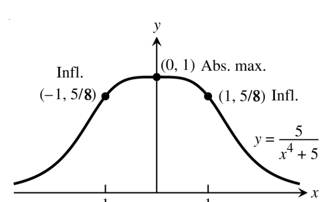
42. When $y = \sqrt[3]{x^3 + 1}$, then $y' = \frac{x^2}{(x^3+1)^{2/3}}$ and $y'' = \frac{2x}{(x^3+1)^{5/3}}$. The curve is risng on $(-\infty, -1)$, $(-1, 0)$, and $(0, \infty)$. There is are no local or absolute extrema. The curve is concave up on $(-\infty, -1)$ and $(0, \infty)$, and concave down on $(-1, 0)$. There are points of inflection at $x = -1$ and $x = 0$.



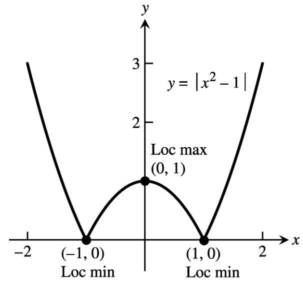
43. When $y = \frac{8x}{x^2 + 4}$, then $y' = \frac{-8(x^2 - 4)}{(x^2+4)^2}$ and $y'' = \frac{16x(x^2 - 12)}{(x^2+4)^3}$. The curve is fallng on $(-\infty, -2)$ and $(2, \infty)$, and is rising on $(-2, 2)$. There is a local and absolute minimum at $x = -2$, and a local and absolute maximum at $x = 2$. The curve is concave down on $\left(-\infty, -2\sqrt{3}\right)$ and $\left(0, 2\sqrt{3}\right)$, and concave up on $\left(-2\sqrt{3}, 0\right)$ and $\left(2\sqrt{3}, \infty\right)$. There are points of inflection at $x = -2\sqrt{3}$, $x = 0$, and $x = 2\sqrt{3}$. $y = 0$ is a horizontal asymptote.



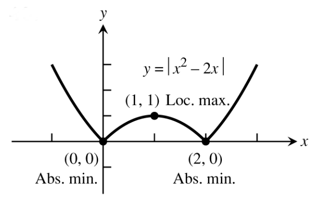
44. When $y = \frac{5}{x^4 + 5}$, then $y' = \frac{-20x^3}{(x^4+5)^2}$ and $y'' = \frac{100x^2(x^4 - 3)}{(x^4+5)^3}$. The curve is risng on $(-\infty, 0)$, and is falling on $(0, \infty)$. There is a local and absolute maximum at $x = 0$, and there is no local or aboslute minimum. The curve is concave up on $\left(-\infty, -\sqrt[4]{3}\right)$ and $\left(\sqrt[4]{3}, \infty\right)$, and concave down on $\left(-\sqrt[4]{3}, 0\right)$ and $\left(0, \sqrt[4]{3}\right)$. There are points of inflection at $x = -\sqrt[4]{3}$ and $x = \sqrt[4]{3}$. There is a horizontal asymptote of $y = 0$.



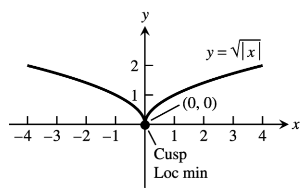
45. When $y = |x^2 - 1| = \begin{cases} x^2 - 1, & |x| \geq 1 \\ 1 - x^2, & |x| < 1 \end{cases}$, then $y' = \begin{cases} 2x, & |x| > 1 \\ -2x, & |x| < 1 \end{cases}$ and $y'' = \begin{cases} 2, & |x| > 1 \\ -2, & |x| < 1 \end{cases}$. The curve rises on $(-1, 0)$ and $(1, \infty)$ and falls on $(-\infty, -1)$ and $(0, 1)$. There is a local maximum at $x = 0$ and local minima at $x = \pm 1$. The curve is concave up on $(-\infty, -1)$ and $(1, \infty)$, and concave down on $(-1, 1)$. There are no points of inflection because y is not differentiable at $x = \pm 1$ (so there is no tangent line at those points).



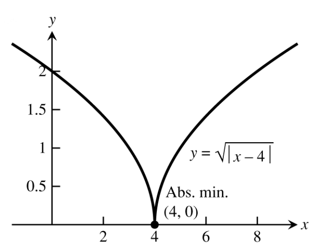
46. When $y = |x^2 - 2x| = \begin{cases} x^2 - 2x, & x < 0 \\ 2x - x^2, & 0 \leq x \leq 2 \\ x^2 - 2x, & x > 2 \end{cases}$, then $y' = \begin{cases} 2x - 2, & x < 0 \\ 2 - 2x, & 0 < x < 2 \\ 2x - 2, & x > 2 \end{cases}$, and $y'' = \begin{cases} 2, & x < 0 \\ -2, & 0 < x < 2 \\ 2, & x > 2 \end{cases}$.
The curve is rising on $(0, 1)$ and $(2, \infty)$, and falling on $(-\infty, 0)$ and $(1, 2)$. There is a local maximum at $x = 1$ and local minima at $x = 0$ and $x = 2$. The curve is concave up on $(-\infty, 0)$ and $(2, \infty)$, and concave down on $(0, 2)$. There are no points of inflection because y is not differentiable at $x = 0$ and $x = 2$ (so there is no tangent at those points).



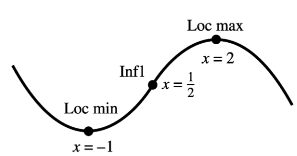
47. When $y = \sqrt{|x|} = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$, then $y' = \begin{cases} \frac{1}{2\sqrt{x}}, & x > 0 \\ \frac{-1}{2\sqrt{-x}}, & x < 0 \end{cases}$ and $y'' = \begin{cases} \frac{-x^{-3/2}}{4}, & x > 0 \\ \frac{-(-x)^{-3/2}}{4}, & x < 0 \end{cases}$.
Since $\lim_{x \to 0^-} y' = -\infty$ and $\lim_{x \to 0^+} y' = \infty$ there is a cusp at $x = 0$. There is a local minimum at $x = 0$, but no local maximum. The curve is concave down on $(-\infty, 0)$ and $(0, \infty)$. There are no points of inflection.



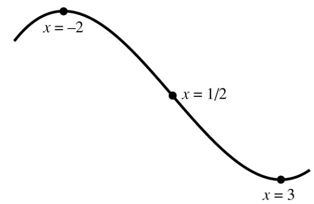
48. When $y = \sqrt{|x - 4|} = \begin{cases} \sqrt{x - 4}, & x \geq 4 \\ \sqrt{4 - x}, & x < 4 \end{cases}$, then $y' = \begin{cases} \frac{1}{2\sqrt{x-4}}, & x > 4 \\ \frac{-1}{2\sqrt{4-x}}, & x < 4 \end{cases}$ and $y'' = \begin{cases} \frac{-(x-4)^{-3/2}}{4}, & x > 4 \\ \frac{-(4-x)^{-3/2}}{4}, & x < 4 \end{cases}$.
Since $\lim_{x \to 4^-} y' = -\infty$ and $\lim_{x \to 4^+} y' = \infty$ there is a cusp at $x = 4$. There is a local minimum at $x = 4$, but no local maximum. The curve is concave down on $(-\infty, 4)$ and $(4, \infty)$. There are no points of inflection.



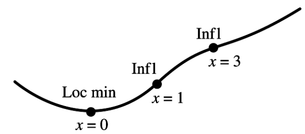
49. $y' = 2 + x - x^2 = (1 + x)(2 - x)$, $y' = \underset{-1}{---|}\underset{2}{+++|}---$
$\Rightarrow$ rising on $(-1, 2)$, falling on $(-\infty, -1)$ and $(2, \infty)$
$\Rightarrow$ there is a local maximum at $x = 2$ and a local minimum at $x = -1$; $y'' = 1 - 2x$, $y'' = \underset{1/2}{+++|}---$
$\Rightarrow$ concave up on $\left(-\infty, \frac{1}{2}\right)$, concave down on $\left(\frac{1}{2}, \infty\right)$ $\Rightarrow$ a point of inflection at $x = \frac{1}{2}$



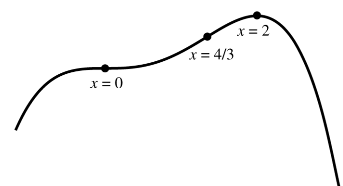
50. $y' = x^2 - x - 6 = (x-3)(x+2)$, $y' = \underset{-2}{+++ \mid} \underset{3}{--- \mid} +++$
$\Rightarrow$ rising on $(-\infty, -2)$ and $(3, \infty)$, falling on $(-2, 3)$
$\Rightarrow$ there is a local maximum at $x = -2$ and a local minimum at $x = 3$; $y'' = 2x - 1$, $y'' = \underset{1/2}{--- \mid} +++$
$\Rightarrow$ concave up on $\left(\frac{1}{2}, \infty\right)$, concave down on $\left(-\infty, \frac{1}{2}\right)$
$\Rightarrow$ a point of inflection at $x = \frac{1}{2}$



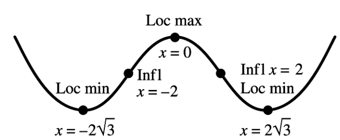
51. $y' = x(x-3)^2$, $y' = \underset{0}{--- \mid} \underset{3}{+++ \mid} +++ \Rightarrow$ rising on $(0, \infty)$, falling on $(-\infty, 0) \Rightarrow$ no local maximum, but there is a local minimum at $x = 0$; $y'' = (x-3)^2 + x(2)(x-3)$ $= 3(x-3)(x-1)$, $y'' = \underset{1}{+++ \mid} \underset{3}{--- \mid} +++ \Rightarrow$ concave up on $(-\infty, 1)$ and $(3, \infty)$, concave down on $(1, 3) \Rightarrow$ points of inflection at $x = 1$ and $x = 3$



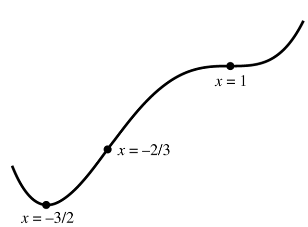
52. $y' = x^2(2 - x)$, $y' = \underset{0}{+++ \mid} \underset{2}{+++ \mid} --- \Rightarrow$ rising on $(-\infty, 2)$, falling on $(2, \infty) \Rightarrow$ there is a local maximum at $x = 2$, but no local minimum; $y'' = 2x(2-x) + x^2(-1)$ $= x(4 - 3x)$, $y'' = \underset{0}{--- \mid} \underset{4/3}{+++ \mid} --- \Rightarrow$ concave up on $\left(0, \frac{4}{3}\right)$, concave down on $(-\infty, 0)$ and $\left(\frac{4}{3}, \infty\right) \Rightarrow$ points of inflection at $x = 0$ and $x = \frac{4}{3}$



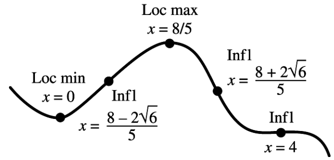
53. $y' = x\left(x^2 - 12\right) = x\left(x - 2\sqrt{3}\right)\left(x + 2\sqrt{3}\right)$,
$y' = \underset{-2\sqrt{3}}{--- \mid} \underset{0}{+++ \mid} \underset{2\sqrt{3}}{--- \mid} +++ \Rightarrow$ rising on $\left(-2\sqrt{3}, 0\right)$ and $\left(2\sqrt{3}, \infty\right)$, falling on $\left(-\infty, -2\sqrt{3}\right)$ and $\left(0, 2\sqrt{3}\right) \Rightarrow$ a local maximum at $x = 0$, local minima at $x = \pm 2\sqrt{3}$; $y'' = 1\left(x^2 - 12\right) + x(2x) = 3(x-2)(x+2)$,
$y'' = \underset{-2}{+++ \mid} \underset{2}{--- \mid} +++ \Rightarrow$ concave up on $(-\infty, -2)$ and $(2, \infty)$, concave down on $(-2, 2) \Rightarrow$ points of inflection at $x = \pm 2$



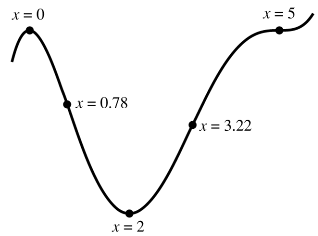
54. $y' = (x-1)^2(2x+3)$, $y' = \underset{-3/2}{--- \mid} \underset{1}{+++ \mid} +++$
$\Rightarrow$ rising on $\left(-\frac{3}{2}, \infty\right)$, falling on $\left(-\infty, -\frac{3}{2}\right) \Rightarrow$ no local maximum, a local minimum at $x = -\frac{3}{2}$;
$y'' = 2(x-1)(2x+3) + (x-1)^2(2) = 2(x-1)(3x+2)$,
$y'' = \underset{-2/3}{+++ \mid} \underset{1}{--- \mid} +++ \Rightarrow$ concave up on $\left(-\infty, -\frac{2}{3}\right)$ and $(1, \infty)$, concave down on $\left(-\frac{2}{3}, 1\right) \Rightarrow$ points of inflection at $x = -\frac{2}{3}$ and $x = 1$



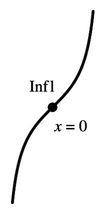
55. $y' = (8x - 5x^2)(4 - x)^2 = x(8 - 5x)(4 - x)^2$,
$y' = ---\underset{0}{|}+++\underset{8/5}{|}---\underset{4}{|}--- \Rightarrow$ rising on $\left(0, \frac{8}{5}\right)$,
falling on $(-\infty, 0)$ and $\left(\frac{8}{5}, \infty\right) \Rightarrow$ a local maximum at
$x = \frac{8}{5}$, a local minimum at $x = 0$;
$y'' = (8 - 10x)(4 - x)^2 + (8x - 5x^2)(2)(4 - x)(-1) = 4(4 - x)(5x^2 - 16x + 8)$,
$y'' = +++\underset{\frac{8-2\sqrt{6}}{5}}{|}---\underset{\frac{8+2\sqrt{6}}{5}}{|}+++\underset{4}{|}--- \Rightarrow$ concave up on $\left(-\infty, \frac{8-2\sqrt{6}}{5}\right)$ and $\left(\frac{8+2\sqrt{6}}{5}, 4\right)$, concave down on $\left(\frac{8-2\sqrt{6}}{5}, \frac{8+2\sqrt{6}}{5}\right)$ and $(4, \infty) \Rightarrow$ points of inflection at $x = \frac{8 \pm 2\sqrt{6}}{5}$ and $x = 4$



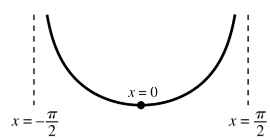
56. $y' = (x^2 - 2x)(x - 5)^2 = x(x - 2)(x - 5)^2$,
$y' = +++\underset{0}{|}---\underset{2}{|}+++\underset{5}{|}+++ \Rightarrow$ rising on $(-\infty, 0)$ and
$(2, \infty)$, falling on $(0, 2) \Rightarrow$ a local maximum at $x = 0$, a local minimum at $x = 2$;
$y'' = (2x - 2)(x - 5)^2 + 2(x^2 - 2x)(x - 5)$
$= 2(x - 5)(2x^2 - 8x + 5)$,
$y'' = ---\underset{\frac{4-\sqrt{6}}{2}}{|}+++\underset{\frac{4+\sqrt{6}}{2}}{|}---\underset{5}{|}+++ \Rightarrow$ concave up on
$\left(\frac{4-\sqrt{6}}{2}, \frac{4+\sqrt{6}}{2}\right)$ and $(5, \infty)$, concave down on $\left(-\infty, \frac{4-\sqrt{6}}{2}\right)$ and $\left(\frac{4+\sqrt{6}}{2}, 5\right) \Rightarrow$ points of inflection at $x = \frac{4 \pm \sqrt{6}}{2}$ and $x = 5$



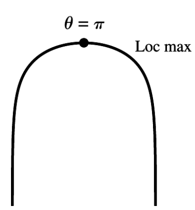
57. $y' = \sec^2 x$, $y' = \underset{-\pi/2}{(}\ +++\ \underset{\pi/2}{)} \Rightarrow$ rising on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,
never falling $\Rightarrow$ no local extrema; $y'' = 2(\sec x)(\sec x)(\tan x)$
$= 2(\sec^2 x)(\tan x)$, $y'' = \underset{-\pi/2}{(}\ ---\underset{0}{|}+++\ \underset{\pi/2}{)} \Rightarrow$ concave
up on $\left(0, \frac{\pi}{2}\right)$, concave down on $\left(-\frac{\pi}{2}, 0\right)$, 0 is a point of inflection.



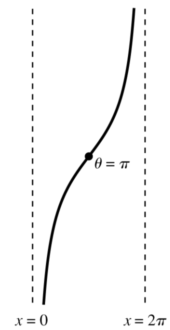
58. $y' = \tan x$, $y' = \underset{-\pi/2}{(}\ ---\underset{0}{|}+++\ \underset{\pi/2}{)} \Rightarrow$ rising on $\left(0, \frac{\pi}{2}\right)$,
falling on $\left(-\frac{\pi}{2}, 0\right) \Rightarrow$ no local maximum, a local minimum
at $x = 0$; $y'' = \sec^2 x$, $y'' = \underset{-\pi/2}{(}\ +++\ \underset{\pi/2}{)} \Rightarrow$ concave up
on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow$ no points of inflection



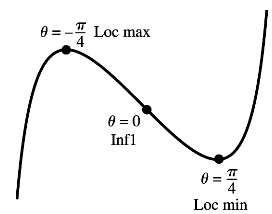
59. $y' = \cot \frac{\theta}{2}$, $y' = \underset{0}{(}+++\underset{\pi}{|}---\underset{2\pi}{)} \Rightarrow$ rising on $(0, \pi)$,
falling on $(\pi, 2\pi) \Rightarrow$ a local maximum at $\theta = \pi$, no local
minimum; $y'' = -\frac{1}{2}\csc^2 \frac{\theta}{2}$, $y'' = \underset{0}{(}---\underset{2\pi}{)} \Rightarrow$ never
concave up, concave down on $(0, 2\pi) \Rightarrow$ no points of inflection



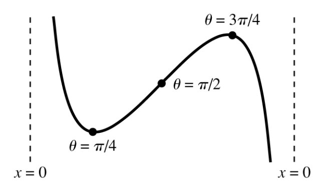
60. $y' = \csc^2 \frac{\theta}{2}$, $y' = \underset{0}{(} +++ \underset{2\pi}{)}$ $\Rightarrow$ rising on $(0, 2\pi)$, never falling $\Rightarrow$ no local extrema;
$y'' = 2\left(\csc \frac{\theta}{2}\right)\left(-\csc \frac{\theta}{2}\right)\left(\cot \frac{\theta}{2}\right)\left(\frac{1}{2}\right)$
$= -\left(\csc^2 \frac{\theta}{2}\right)\left(\cot \frac{\theta}{2}\right)$, $y'' = \underset{0}{(} --- \underset{\pi}{|} +++ \underset{2\pi}{)}$
$\Rightarrow$ concave up on $(\pi, 2\pi)$, concave down on $(0, \pi)$
$\Rightarrow$ a point of inflection at $\theta = \pi$



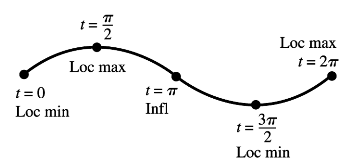
61. $y' = \tan^2 \theta - 1 = (\tan \theta - 1)(\tan \theta + 1)$,
$y' = \underset{-\pi/2}{(} +++ \underset{-\pi/4}{|} --- \underset{\pi/4}{|} +++ \underset{\pi/2}{)}$ $\Rightarrow$ rising on $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, falling on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
$\Rightarrow$ a local maximum at $\theta = -\frac{\pi}{4}$, a local minimum at $\theta = \frac{\pi}{4}$;
$y'' = 2 \tan \theta \sec^2 \theta$, $y'' = \underset{-\pi/2}{(} --- \underset{0}{|} +++ \underset{\pi/2}{)}$
$\Rightarrow$ concave up on $\left(0, \frac{\pi}{2}\right)$, concave down on $\left(-\frac{\pi}{2}, 0\right)$
$\Rightarrow$ a point of inflection at $\theta = 0$



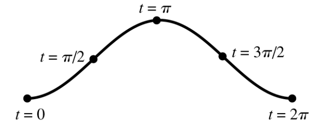
62. $y' = 1 - \cot^2 \theta = (1 - \cot \theta)(1 + \cot \theta)$,
$y' = \underset{0}{(} --- \underset{\pi/4}{|} +++ \underset{3\pi/4}{|} --- \underset{\pi}{)}$ $\Rightarrow$ rising on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, falling on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$ $\Rightarrow$ a local maximum at $\theta = \frac{3\pi}{4}$, a local minimum at $\theta = \frac{\pi}{4}$;
$y'' = -2(\cot \theta)(-\csc^2 \theta)$, $y'' = \underset{0}{(} +++ \underset{\pi/2}{|} --- \underset{\pi}{)}$
$\Rightarrow$ concave up on $\left(0, \frac{\pi}{2}\right)$, concave down on $\left(\frac{\pi}{2}, \pi\right)$
$\Rightarrow$ a point of inflection at $\theta = \frac{\pi}{2}$



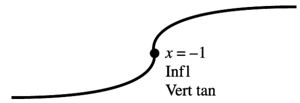
63. $y' = \cos t$, $y' = \underset{0}{[} +++ \underset{\pi/2}{|} --- \underset{3\pi/2}{|} +++ \underset{2\pi}{]}$ $\Rightarrow$ rising on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$, falling on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ $\Rightarrow$ local maxima at $t = \frac{\pi}{2}$ and $t = 2\pi$, local minima at $t = 0$ and $t = \frac{3\pi}{2}$;
$y'' = -\sin t$, $y'' = \underset{0}{[} --- \underset{\pi}{|} +++ \underset{2\pi}{]}$
$\Rightarrow$ concave up on $(\pi, 2\pi)$, concave down on $(0, \pi)$ $\Rightarrow$ a point of inflection at $t = \pi$



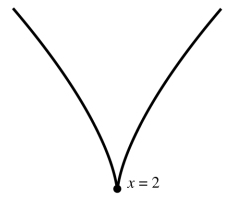
64. $y' = \sin t$, $y' = \underset{0}{[} +++ \underset{\pi}{|} --- \underset{2\pi}{]}$ $\Rightarrow$ rising on $(0, \pi)$, falling on $(\pi, 2\pi)$ $\Rightarrow$ a local maximum at $t = \pi$, local minima at $t = 0$ and $t = 2\pi$; $y'' = \cos t$,
$y'' = \underset{0}{[} +++ \underset{\pi/2}{|} --- \underset{3\pi/2}{|} +++ \underset{2\pi}{]}$ $\Rightarrow$ concave up on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$, concave down on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ $\Rightarrow$ points of inflection at $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$



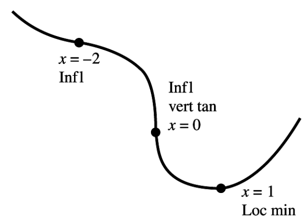
65. $y' = (x+1)^{-2/3}$, $y' = +++\underset{-1}{)}(+++ \Rightarrow$ rising on $(-\infty, \infty)$, never falling $\Rightarrow$ no local extrema; $y'' = -\frac{2}{3}(x+1)^{-5/3}$, $y'' = +++\underset{-1}{)}(---$ $\Rightarrow$ concave up on $(-\infty, -1)$, concave down on $(-1, \infty)$ $\Rightarrow$ a point of inflection and vertical tangent at $x = -1$



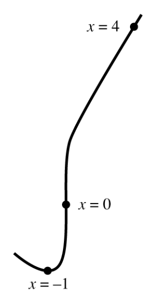
66. $y' = (x-2)^{-1/3}$, $y' = ---\underset{2}{)}(+++ \Rightarrow$ rising on $(2, \infty)$, falling on $(-\infty, 2)$ $\Rightarrow$ no local maximum, but a local minimum at $x = 2$; $y'' = -\frac{1}{3}(x-2)^{-4/3}$, $y'' = ---\underset{2}{)}(--- \Rightarrow$ concave down on $(-\infty, 2)$ and $(2, \infty)$ $\Rightarrow$ no points of inflection, but there is a cusp at $x = 2$



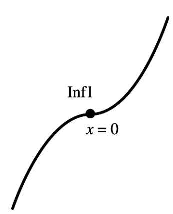
67. $y' = x^{-2/3}(x-1)$, $y' = ---\underset{0}{)}(---\underset{1}{|}+++ \Rightarrow$ rising on $(1, \infty)$, falling on $(-\infty, 1)$ $\Rightarrow$ no local maximum, but a local minimum at $x = 1$; $y'' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$ $= \frac{1}{3}x^{-5/3}(x+2)$, $y'' = +++\underset{-2}{|}---\underset{0}{)}(+++$ $\Rightarrow$ concave up on $(-\infty, -2)$ and $(0, \infty)$, concave down on $(-2, 0)$ $\Rightarrow$ points of inflection at $x = -2$ and $x = 0$, and a vertical tangent at $x = 0$



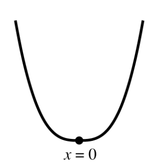
68. $y' = x^{-4/5}(x+1)$, $y' = ---\underset{-1}{|}+++\underset{0}{)}(+++ \Rightarrow$ rising on $(-1, 0)$ and $(0, \infty)$, falling on $(-\infty, -1)$ $\Rightarrow$ no local maximum, but a local minimum at $x = -1$; $y'' = \frac{1}{5}x^{-4/5} - \frac{4}{5}x^{-9/5} = \frac{1}{5}x^{-9/5}(x-4)$, $y'' = +++\underset{0}{)}(---\underset{4}{|}+++ \Rightarrow$ concave up on $(-\infty, 0)$ and $(4, \infty)$, concave down on $(0, 4)$ $\Rightarrow$ points of inflection at $x = 0$ and $x = 4$, and a vertical tangent at $x = 0$



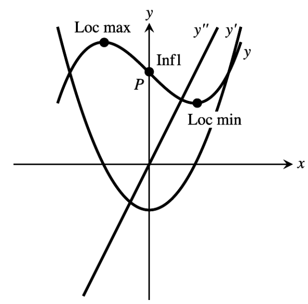
69. $y' = \begin{cases} -2x, & x \le 0 \\ 2x, & x > 0 \end{cases}$, $y' = +++\underset{0}{|}+++ \Rightarrow$ rising on $(-\infty, \infty) \Rightarrow$ no local extrema; $y'' = \begin{cases} -2, & x < 0 \\ 2, & x > 0 \end{cases}$, $y'' = ---\underset{0}{)}(+++ \Rightarrow$ concave up on $(0, \infty)$, concave down on $(-\infty, 0)$ $\Rightarrow$ a point of inflection at $x = 0$



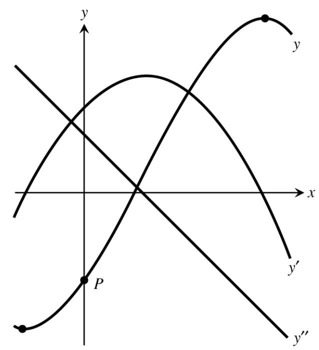
70. $y' = \begin{cases} -x^2, & x \le 0 \\ x^2, & x > 0 \end{cases}$, $y' = ---\underset{0}{|}+++ \Rightarrow$ rising on $(0, \infty)$, falling on $(-\infty, 0)$ $\Rightarrow$ no local maximum, but a local minimum at $x = 0$; $y'' = \begin{cases} -2x, & x \le 0 \\ 2x, & x > 0 \end{cases}$, $y'' = +++\underset{0}{|}+++ \Rightarrow$ concave up on $(-\infty, \infty)$ $\Rightarrow$ no point of inflection



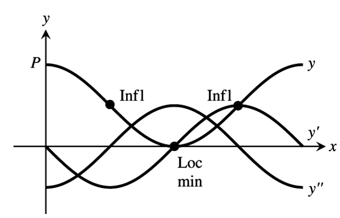
71. The graph of $y = f''(x) \Rightarrow$ the graph of $y = f(x)$ is concave up on $(0, \infty)$, concave down on $(-\infty, 0) \Rightarrow$ a point of inflection at $x = 0$; the graph of $y = f'(x)$ $\Rightarrow y' = +++ \mid --- \mid +++ \Rightarrow$ the graph $y = f(x)$ has both a local maximum and a local minimum



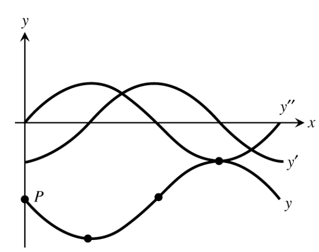
72. The graph of $y = f''(x) \Rightarrow y'' = +++ \mid --- \Rightarrow$ the graph of $y = f(x)$ has a point of inflection, the graph of $y = f'(x) \Rightarrow y' = --- \mid +++ \mid --- \Rightarrow$ the graph of $y = f(x)$ has both a local maximum and a local minimum



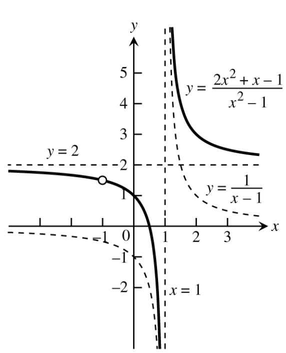
73. The graph of $y = f''(x) \Rightarrow y'' = --- \mid +++ \mid ---$ $\Rightarrow$ the graph of $y = f(x)$ has two points of inflection, the graph of $y = f'(x) \Rightarrow y' = --- \mid +++ \Rightarrow$ the graph of $y = f(x)$ has a local minimum



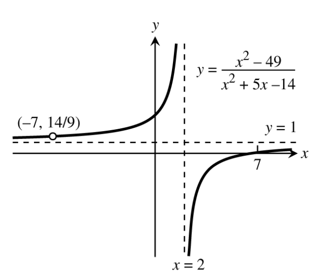
74. The graph of $y = f''(x) \Rightarrow y'' = +++ \mid --- \Rightarrow$ the graph of $y = f(x)$ has a point of inflection; the graph of $y = f'(x) \Rightarrow y' = --- \mid +++ \mid --- \Rightarrow$ the graph of $y = f(x)$ has both a local maximum and a local minimum



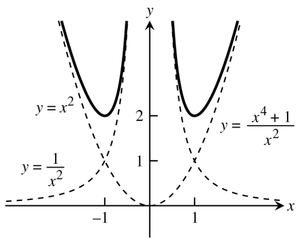
75. $y = \frac{2x^2 + x - 1}{x^2 - 1}$



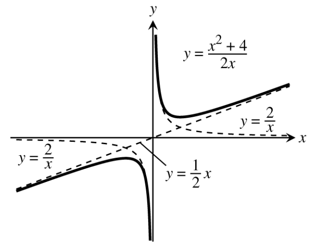
76. $y = \frac{x^2 - 49}{x^2 + 5x - 14} = 1 - \frac{5}{x - 2}$



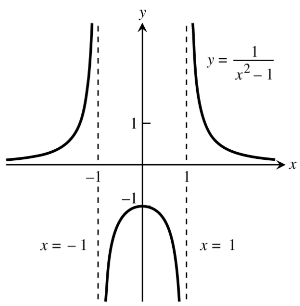
77. $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2}$



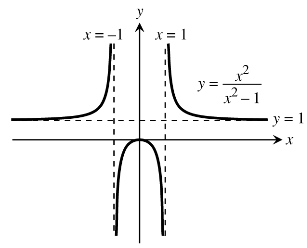
78. $y = \frac{x^2+4}{2x} = \frac{x}{2} + \frac{2}{x}$



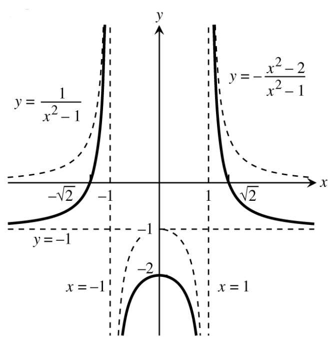
79. $y = \frac{1}{x^2-1}$



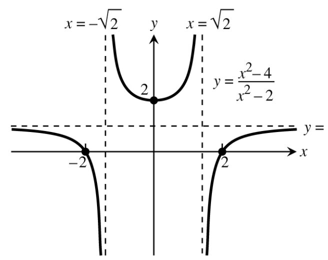
80. $y = \frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1}$



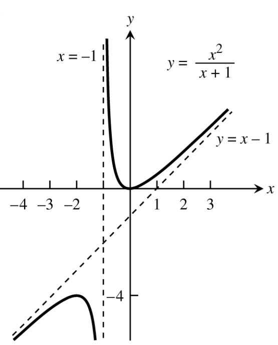
81. $y = -\frac{x^2-2}{x^2-1} = -1 + \frac{1}{x^2-1}$



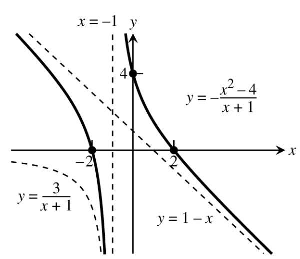
82. $y = \frac{x^2-4}{x^2-2} = 1 - \frac{2}{x^2-2}$



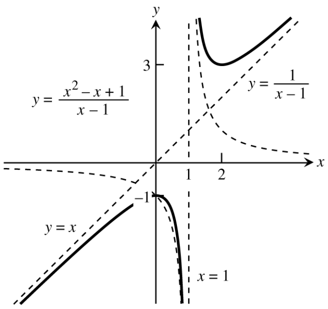
83. $y = \frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$



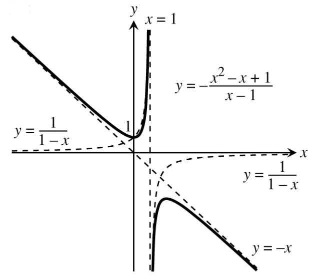
84. $y = -\frac{x^2-4}{x+1} = 1 - x + \frac{3}{x+1}$



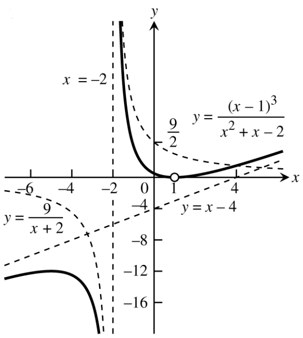
85. $y = \frac{x^2 - x + 1}{x - 1} = x + \frac{1}{x - 1}$



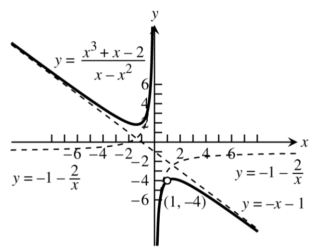
86. $y = -\frac{x^2 - x + 1}{x - 1} = -x - \frac{1}{x - 1}$



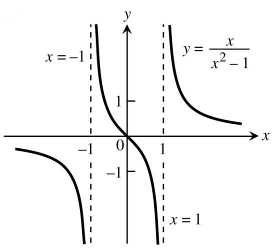
87. $y = \frac{x^3 - 3x^2 + 3x - 1}{x^2 + x + 2} = x - 4 + \frac{5x + 7}{x^2 + x + 2}$



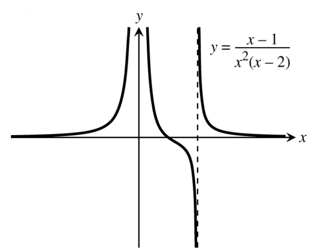
88. $y = \frac{x^3 + x - 2}{x - x^2} = -x - 1 + \frac{2x - 2}{x - x^2}$



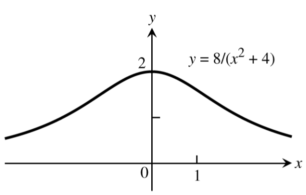
89. $y = \frac{x}{x^2 - 1}$



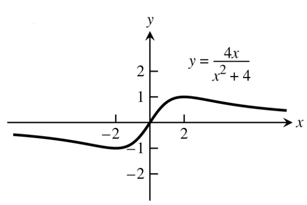
90. $y = \frac{x - 1}{x^2(x - 2)}$



91. $y = \frac{8}{x^2 + 4}$



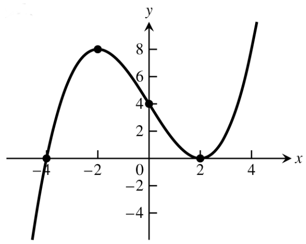
92. $y = \frac{4x}{x^2 + 4}$



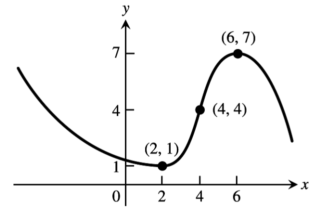
93.

Point	y'	y''
P	$-$	$+$
Q	$+$	0
R	$+$	$-$
S	0	$-$
T	$-$	$-$

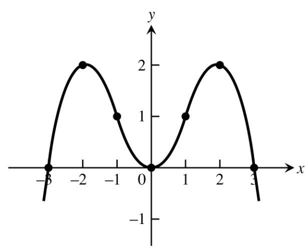
94.



95.



96.



97. Graphs printed in color can shift during a press run, so your values may differ somewhat from those given here.
   (a) The body is moving away from the origin when |displacement| is increasing as t increases, $0 < t < 2$ and $6 < t < 9.5$; the body is moving toward the origin when |displacement| is decreasing as t increases, $2 < t < 6$ and $9.5 < t < 15$
   (b) The velocity will be zero when the slope of the tangent line for $y = s(t)$ is horizontal. The velocity is zero when t is approximately 2, 6, or 9.5 sec.
   (c) The acceleration will be zero at those values of t where the curve $y = s(t)$ has points of inflection. The acceleration is zero when t is approximately 4, 7.5, or 12.5 sec.
   (d) The acceleration is positive when the concavity is up, $4 < t < 7.5$ and $12.5 < t < 15$; the acceleration is negative when the concavity is down, $0 < t < 4$ and $7.5 < t < 12.5$

98. (a) The body is moving away from the origin when |displacement| is increasing as t increases, $1.5 < t < 4$, $10 < t < 12$ and $13.5 < t < 16$; the body is moving toward the origin when |displacement| is decreasing as t increases, $0 < t < 1.5$, $4 < t < 10$ and $12 < t < 13.5$
   (b) The velocity will be zero when the slope of the tangent line for $y = s(t)$ is horizontal. The velocity is zero when t is approximately 0, 4, 12 or 16 sec.
   (c) The acceleration will be zero at those values of t where the curve $y = s(t)$ has points of inflection. The acceleration is zero when t is approximately 1.5, 6, 8, 10.5, or 13.5 sec.
   (d) The acceleration is positive when the concavity is up, $0 < t < 1.5$, $6 < t < 8$ and $10 < t < 13.5$, the acceleration is negative when the concavity is down, $1.5 < t < 6$, $8 < t < 10$ and $13.5 < t < 16$.

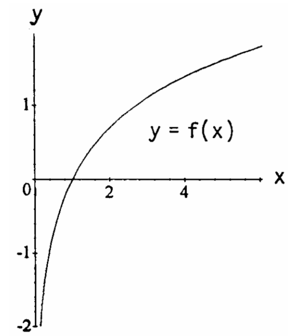
99. The marginal cost is $\frac{dc}{dx}$ which changes from decreasing to increasing when its derivative $\frac{d^2c}{dx^2}$ is zero. This is a point of inflection of the cost curve and occurs when the production level x is approximately 60 thousand units.

100. The marginal revenue is $\frac{dy}{dx}$ and it is increasing when its derivative $\frac{d^2y}{dx^2}$ is positive $\Rightarrow$ the curve is concave up $\Rightarrow$ $0 < t < 2$ and $5 < t < 9$; marginal revenue is decreasing when $\frac{d^2y}{dx^2} < 0$ $\Rightarrow$ the curve is concave down $\Rightarrow$ $2 < t < 5$ and $9 < t < 12$.

101. When $y' = (x-1)^2(x-2)$, then $y'' = 2(x-1)(x-2) + (x-1)^2$. The curve falls on $(-\infty, 2)$ and rises on $(2, \infty)$. At $x = 2$ there is a local minimum. There is no local maximum. The curve is concave upward on $(-\infty, 1)$ and $\left(\frac{5}{3}, \infty\right)$, and concave downward on $\left(1, \frac{5}{3}\right)$. At $x = 1$ or $x = \frac{5}{3}$ there are inflection points.

102. When $y' = (x-1)^2(x-2)(x-4)$, then $y'' = 2(x-1)(x-2)(x-4) + (x-1)^2(x-4) + (x-1)^2(x-2)$ $= (x-1)\left[2\left(x^2-6x+8\right) + \left(x^2-5x+4\right) + \left(x^2-3x+2\right)\right] = 2(x-1)\left(2x^2-10x+11\right)$. The curve rises on $(-\infty, 2)$ and $(4, \infty)$ and falls on $(2, 4)$. At $x = 2$ there is a local maximum and at $x = 4$ a local minimum. The curve is concave downward on $(-\infty, 1)$ and $\left(\frac{5-\sqrt{3}}{2}, \frac{5+\sqrt{3}}{2}\right)$ and concave upward on $\left(1, \frac{5-\sqrt{3}}{2}\right)$ and $\left(\frac{5+\sqrt{3}}{2}, \infty\right)$. At $x = 1$, $\frac{5-\sqrt{3}}{2}$ and $\frac{5+\sqrt{3}}{2}$ there are inflection points.

103. The graph must be concave down for $x > 0$ because $f''(x) = -\frac{1}{x^2} < 0$.



104. The second derivative, being continuous and never zero, cannot change sign. Therefore the graph will always be concave up or concave down so it will have no inflection points and no cusps or corners.

105. The curve will have a point of inflection at $x = 1$ if 1 is a solution of $y'' = 0$; $y = x^3 + bx^2 + cx + d$ $\Rightarrow\ y' = 3x^2 + 2bx + c\ \Rightarrow\ y'' = 6x + 2b$ and $6(1) + 2b = 0\ \Rightarrow\ b = -3$.

106. (a) $f(x) = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a}$ a parabola whose vertex is at $x = -\frac{b}{2a}\ \Rightarrow$ the coordinates of the vertex are $\left(-\frac{b}{2a}, -\frac{b^2-4ac}{4a}\right)$

(b) The second derivative, $f''(x) = 2a$, describes concavity $\Rightarrow$ when $a > 0$ the parabola is concave up and when $a < 0$ the parabola is concave down.

107. A quadratic curve never has an inflection point. If $y = ax^2 + bx + c$ where $a \neq 0$, then $y' = 2ax + b$ and $y'' = 2a$. Since $2a$ is a constant, it is not possible for y'' to change signs.

108. A cubic curve always has exactly one inflection point. If $y = ax^3 + bx^2 + cx + d$ where $a \neq 0$, then $y' = 3ax^2 + 2bx + c$ and $y'' = 6ax + 2b$. Since $\frac{-b}{3a}$ is a solution of $y'' = 0$, we have that y'' changes its sign at $x = -\frac{b}{3a}$ and y' exists everywhere (so there is a tangent at $x = -\frac{b}{3a}$). Thus the curve has an inflection point at $x = -\frac{b}{3a}$. There are no other inflection points because y'' changes sign only at this zero.

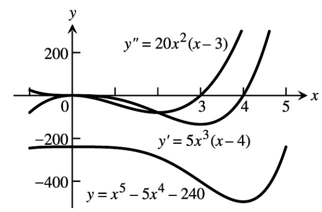
109. $y'' = (x+1)(x-2)$, when $y'' = 0 \Rightarrow x = -1$ or $x = 2$; $y'' = +++ \underset{-1}{|} --- \underset{2}{|} +++ \Rightarrow$ points of inflection at $x = -1$ and $x = 2$

110. $y'' = x^2(x-2)^3(x+3)$, when $y'' = 0 \Rightarrow x = -3$, $x = 0$, or $x = 2$; $y'' = +++ \underset{-3}{|} --- \underset{0}{|} --- \underset{2}{|} +++ \Rightarrow$ points of inflection at $x = -3$ and $x = 2$

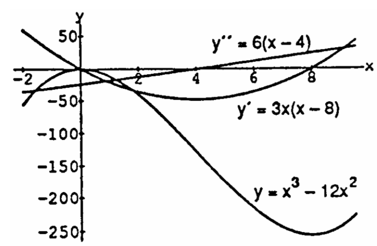
111. $y = ax^3 + bx^2 + cx \Rightarrow y' = 3ax^2 + 2bx + c$ and $y'' = 6ax + 2b$; local maximum at $x = 3$ $\Rightarrow 3a(3)^2 + 2b(3) + c = 0 \Rightarrow 27a + 6b + c = 0$; local mimimum at $x = -1 \Rightarrow 3a(-1)^2 + 2b(-1) + c = 0$ $\Rightarrow 3a - 2b + c = 0$; point of inflection at $(1, 11) \Rightarrow a(1)^3 + b(1)^2 + c(1) = 11 \Rightarrow a + b + c = 11$ and $6a(1) + 2b = 0 \Rightarrow 6a + 2b = 0$. Solving $27a + 6b + c = 0$, $3a - 2b + c = 0$, $a + b + c = 11$, and $6a + 2b = 0$ $\Rightarrow a = -1, b = 3$, and $c = 9 \Rightarrow y = -x^3 + 3x^2 + 9x$

112. $y = \frac{x^2+a}{bx+c} \Rightarrow y' = \frac{bx^2+2cx-ab}{(bx+c)^2}$; local maximum at $x = 3 \Rightarrow \frac{b(3)^2+2c(3)-ab}{(b(3)+c)^2} = 0 \Rightarrow 9b + 6c - ab = 0$; local minimum at $(-1, -2) \Rightarrow \frac{b(-1)^2+2c(-1)-ab}{(b(-1)+c)^2} = 0 \Rightarrow b - 2c - ab = 0$ and $\frac{(-1)^2+a}{b(-1)+c} = -2 \Rightarrow -a + 2b - 2c = 1$. Solving $9b + 6c - ab = 0$, $b - 2c - ab = 0$, and $-a + 2b - 2c = 1 \Rightarrow a = 3, b = 1$, and $c = -1 \Rightarrow y = \frac{x^2+3}{x-1}$.

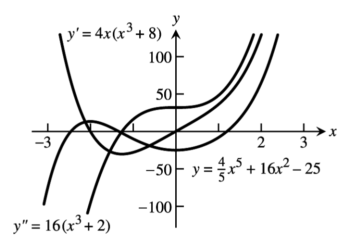
113. If $y = x^5 - 5x^4 - 240$, then $y' = 5x^3(x-4)$ and $y'' = 20x^2(x-3)$. The zeros of y' are extrema, and there is a point of inflection at $x = 3$.



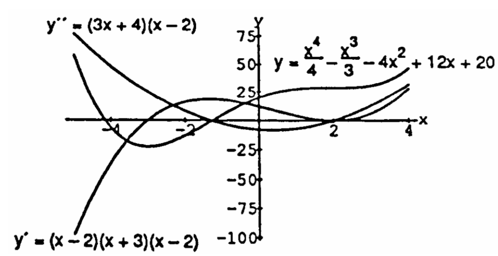
114. If $y = x^3 - 12x^2$, then $y' = 3x(x-8)$ and $y'' = 6(x-4)$. The zeros of y' and y'' are extrema and points of inflection, respectively.



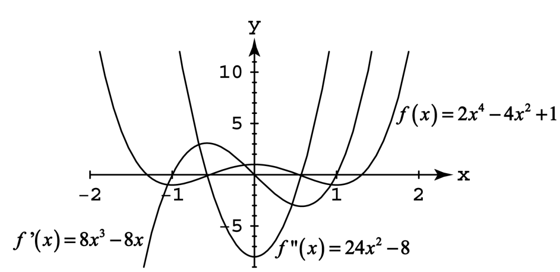
115. If $y = \frac{4}{5}x^5 + 16x^2 - 25$, then $y' = 4x\left(x^3+8\right)$ and $y'' = 16\left(x^3+2\right)$. The zeros of y' and y'' are extrema and points of inflection, respectively.



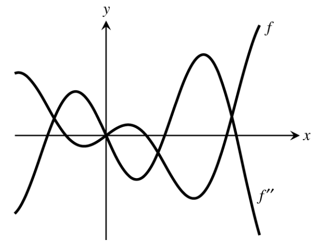
116. If $y = \frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x + 20$, then $y' = x^3 - x^2 - 8x + 12 = (x+3)(x-2)^2$. So y has a local minimum at $x = -3$ as its only extreme value. Also $y'' = 3x^2 - 2x - 8 = (3x+4)(x-2)$ and there are inflection points at both zeros, $-\frac{4}{3}$ and 2, of y''.



117. The graph of f falls where $f' < 0$, rises where $f' > 0$, and has horizontal tangents where $f' = 0$. It has local minima at points where f' changes from negative to positive and local maxima where f' changes from positive to negative. The graph of f is concave down where $f'' < 0$ and concave up where $f'' > 0$. It has an inflection point each time f''changes sign, provided a tangent line exists there.



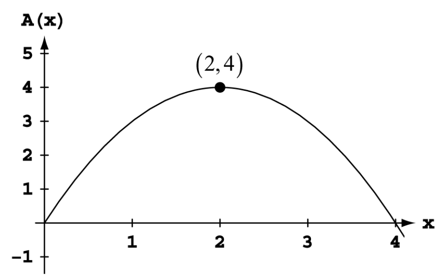
118. The graph f is concave down where $f'' < 0$, and concave up where $f'' > 0$. It has an inflection point each time f'' changes sign, provided a tangent line exists there.



4.5 APPLIED OPTIMIZATION

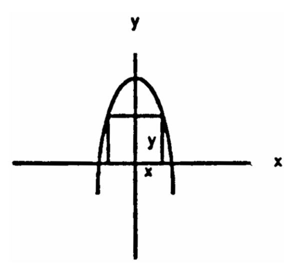
1. Let ℓ and w represent the length and width of the rectangle, respectively. With an area of 16 in.^{2}, we have that $(\ell)(w) = 16 \Rightarrow w = 16\ell^{-1} \Rightarrow$ the perimeter is $P = 2\ell + 2w = 2\ell + 32\ell^{-1}$ and $P'(\ell) = 2 - \frac{32}{\ell^2} = \frac{2(\ell^2 - 16)}{\ell^2}$. Solving $P'(\ell) = 0 \Rightarrow \frac{2(\ell+4)(\ell-4)}{\ell^2} = 0 \Rightarrow \ell = -4, 4$. Since $\ell > 0$ for the length of a rectangle, ℓ must be 4 and $w = 4 \Rightarrow$ the perimeter is 16 in., a minimum since $P''(\ell) = \frac{16}{\ell^3} > 0$.

2. Let x represent the length of the rectangle in meters $(0 < x < 4)$ Then the width is $4 - x$ and the area is $A(x) = x(4 - x) = 4x - x^2$. Since $A'(x) = 4 - 2x$, the critical point occurs at $x = 2$. Since, $A'(x) > 0$ for $0 < x < 2$ and $A'(x) < 0$ for $2 < x < 4$, this critical point corresponds to the maximum area. The rectangle with the largest area measures 2 m by $4 - 2 = 2$ m, so it is a square.
Graphical Support:

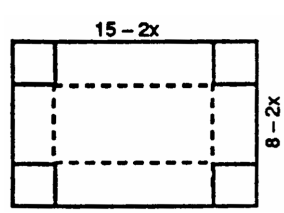


3. (a) The line containing point P also contains the points $(0, 1)$ and $(1, 0)$ $\Rightarrow$ the line containing P is $y = 1 - x$ $\Rightarrow$ a general point on that line is $(x, 1 - x)$.
   (b) The area $A(x) = 2x(1 - x)$, where $0 \le x \le 1$.
   (c) When $A(x) = 2x - 2x^2$, then $A'(x) = 0 \Rightarrow 2 - 4x = 0 \Rightarrow x = \frac{1}{2}$. Since $A(0) = 0$ and $A(1) = 0$, we conclude that $A\left(\frac{1}{2}\right) = \frac{1}{2}$ sq units is the largest area. The dimensions are 1 unit by $\frac{1}{2}$ unit.

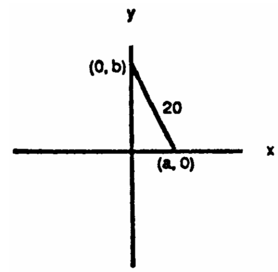
4. The area of the rectangle is $A = 2xy = 2x\left(12 - x^2\right)$, where $0 \le x \le \sqrt{12}$. Solving $A'(x) = 0 \Rightarrow 24 - 6x^2 = 0$ $\Rightarrow x = -2$ or 2. Now -2 is not in the domain, and since $A(0) = 0$ and $A\left(\sqrt{12}\right) = 0$, we conclude that $A(2) = 32$ square units is the maximum area. The dimensions are 4 units by 8 units.



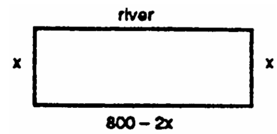
5. The volume of the box is $V(x) = x(15 - 2x)(8 - 2x)$ $= 120x - 46x^2 + 4x^3$, where $0 \le x \le 4$. Solving $V'(x) = 0$ $\Rightarrow 120 - 92x + 12x^2 = 4(6 - x)(5 - 3x) = 0 \Rightarrow x = \frac{5}{3}$ or 6, but 6 is not in the domain. Since $V(0) = V(4) = 0$, $V\left(\frac{5}{3}\right) = \frac{2450}{27} \approx 91$ in^3 must be the maximum volume of the box with dimensions $\frac{14}{3} \times \frac{35}{3} \times \frac{5}{3}$ inches.



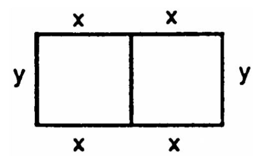
6. The area of the triangle is $A = \frac{1}{2}ba = \frac{b}{2}\sqrt{400 - b^2}$, where $0 \le b \le 20$. Then $\frac{dA}{db} = \frac{1}{2}\sqrt{400 - b^2} - \frac{b^2}{2\sqrt{400 - b^2}}$ $= \frac{200 - b^2}{\sqrt{400 - b^2}} = 0 \Rightarrow$ the interior critical point is $b = 10\sqrt{2}$. When $b = 0$ or 20, the area is zero $\Rightarrow A\left(10\sqrt{2}\right)$ is the maximum area. When $a^2 + b^2 = 400$ and $b = 10\sqrt{2}$, the value of a is also $10\sqrt{2} \Rightarrow$ the maximum area occurs when $a = b$.



7. The area is $A(x) = x(800 - 2x)$, where $0 \le x \le 400$. Solving $A'(x) = 800 - 4x = 0 \Rightarrow x = 200$. With $A(0) = A(400) = 0$, the maximum area is $A(200) = 80{,}000\ m^2$. The dimensions are 200 m by 400 m.



8. The area is $2xy = 216 \Rightarrow y = \frac{108}{x}$. The amount of fence needed is $P = 4x + 3y = 4x + 324x^{-1}$, where $0 < x$; $\frac{dP}{dx} = 4 - \frac{324}{x^2} = 0 \Rightarrow x^2 - 81 = 0 \Rightarrow$ the critical points are 0 and ± 9, but 0 and -9 are not in the domain. Then $P''(9) > 0 \Rightarrow$ at $x = 9$ there is a minimum $\Rightarrow$ the dimensions of the outer rectangle are 18 m by 12 m $\Rightarrow$ 72 meters of fence will be needed.



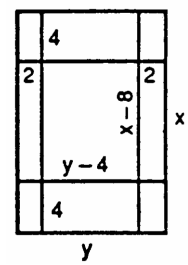
9. (a) We minimize the weight $= tS$ where S is the surface area, and t is the thickness of the steel walls of the tank. The surface area is $S = x^2 + 4xy$ where x is the length of a side of the square base of the tank, and y is its depth. The volume of the tank must be $500 ft^3 \Rightarrow y = \frac{500}{x^2}$. Therefore, the weight of the tank is $w(x) = t\left(x^2 + \frac{2000}{x}\right)$. Treating the thickness as a constant gives $w'(x) = t\left(2x - \frac{2000}{x^2}\right)$. The critical value is at $x = 10$. Since $w''(10) = t\left(2 + \frac{4000}{10^3}\right) > 0$, there is a minimum at $x = 10$. Therefore, the optimum dimensions of the tank are 10 ft on the base edges and 5 ft deep.

(b) Minimizing the surface area of the tank minimizes its weight for a given wall thickness. The thickness of the steel walls would likely be determined by other considerations such as structural requirements.

10. (a) The volume of the tank being $1125\ ft^3$, we have that $yx^2 = 1125 \Rightarrow y = \frac{1125}{x^2}$. The cost of building the tank is $c(x) = 5x^2 + 30x\left(\frac{1125}{x^2}\right)$, where $0 < x$. Then $c'(x) = 10x - \frac{33750}{x^2} = 0 \Rightarrow$ the critical points are 0 and 15, but 0 is not in the domain. Thus, $c''(15) > 0 \Rightarrow$ at $x = 15$ we have a minimum. The values of $x = 15$ ft and $y = 5$ ft will minimize the cost.

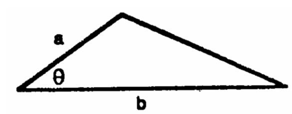
(b) The cost function $c = 5(x^2 + 4xy) + 10xy$, can be separated into two items: (1) the cost of the materials and labor to fabricate the tank, and (2) the cost for the excavation. Since the area of the sides and bottom of the tanks is $(x^2 + 4xy)$, it can be deduced that the unit cost to fabricate the tanks is $\$5/ft^2$. Normally, excavation costs are per unit volume of excavated material. Consequently, the total excavation cost can be taken as $10xy = \left(\frac{10}{x}\right)(x^2y)$. This suggests that the unit cost of excavation is $\frac{\$10/ft^2}{x}$ where x is the length of a side of the square base of the tank in feet. For the least expensive tank, the unit cost for the excavation is $\frac{\$10/ft^2}{15\ ft} = \frac{\$0.67}{ft^3} = \frac{\$18}{yd^3}$. The total cost of the least expensive tank is \$3375, which is the sum of \$2625 for fabrication and \$750 for the excavation.

11. The area of the printing is $(y-4)(x-8)=50$. Consequently, $y=\left(\frac{50}{x-8}\right)+4$. The area of the paper is $A(x)=x\left(\frac{50}{x-8}+4\right)$, where $8<x$. Then $A'(x)=\left(\frac{50}{x-8}+4\right)-x\left(\frac{50}{(x-8)^2}\right)=\frac{4(x-8)^2-400}{(x-8)^2}=0$ $\Rightarrow$ the critical points are -2 and 18, but -2 is not in the domain. Thus $A''(18)>0 \Rightarrow$ at $x=18$ we have a minimum Therefore the dimensions 18 by 9 inches minimize the amount minimize the amount of paper.

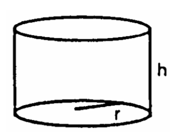


12. The volume of the cone is $V=\frac{1}{3}\pi r^2 h$, where $r=x=\sqrt{9-y^2}$ and $h=y+3$ (from the figure in the text). Thus, $V(y)=\frac{\pi}{3}\left(9-y^2\right)(y+3)=\frac{\pi}{3}\left(27+9y-3y^2-y^3\right) \Rightarrow V'(y)=\frac{\pi}{3}\left(9-6y-3y^2\right)=\pi(1-y)(3+y)$. The critical points are -3 and 1, but -3 is not in the domain. Thus $V''(1)=\frac{\pi}{3}(-6-6(1))<0 \Rightarrow$ at $y=1$ we have a maximum volume of $V(1)=\frac{\pi}{3}(8)(4)=\frac{32\pi}{3}$ cubic units.

13. The area of the triangle is $A(\theta)=\frac{ab\sin\theta}{2}$, where $0<\theta<\pi$. Solving $A'(\theta)=0 \Rightarrow \frac{ab\cos\theta}{2}=0 \Rightarrow \theta=\frac{\pi}{2}$. Since $A''(\theta)=-\frac{ab\sin\theta}{2} \Rightarrow A''\left(\frac{\pi}{2}\right)<0$, there is a maximum at $\theta=\frac{\pi}{2}$.

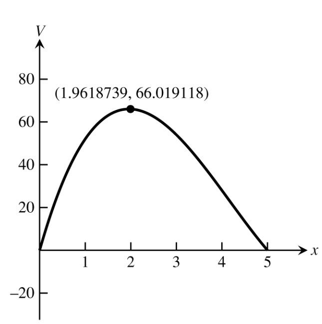


14. A volume $V=\pi r^2 h=1000 \Rightarrow h=\frac{1000}{\pi r^2}$. The amount of material is the surface area given by the sides and bottom of the can $\Rightarrow S=2\pi rh+\pi r^2=\frac{2000}{r}+\pi r^2$, $0<r$. Then $\frac{dS}{dr}=-\frac{2000}{r^2}+2\pi r=0 \Rightarrow \frac{\pi r^3-1000}{r^2}=0$. The critical points are 0 and $\frac{10}{\sqrt[3]{\pi}}$, but 0 is not in the domain. Since $\frac{d^2S}{dr^2}=\frac{4000}{r^3}+2\pi>0$, we have a minimum surface area when $r=\frac{10}{\sqrt[3]{\pi}}$ cm and $h=\frac{1000}{\pi r^2}=\frac{10}{\sqrt[3]{\pi}}$ cm. Comparing this result to the result found in Example 2, if we include both ends of the can, then we have a minimum surface area when the can is shorter-specifically, when the height of the can is the same as its diameter.



15. With a volume of 1000 cm and $V=\pi r^2 h$, then $h=\frac{1000}{\pi r^2}$. The amount of aluminum used per can is $A=8r^2+2\pi rh=8r^2+\frac{2000}{r}$. Then $A'(r)=16r-\frac{2000}{r^2}=0 \Rightarrow \frac{8r^3-1000}{r^2}=0 \Rightarrow$ the critical points are 0 and 5, but $r=0$ results in no can. Since $A''(r)=16+\frac{1000}{r^3}>0$ we have a minimum at $r=5 \Rightarrow h=\frac{40}{\pi}$ and $h{:}r=8{:}\pi$.

16. (a) The base measures $10-2x$ in. by $\frac{15-2x}{2}$ in., so the volume formula is $V(x)=\frac{x(10-2x)(15-2x)}{2}=2x^3-25x^2+75x$.
    (b) We require $x>0$, $2x<10$, and $2x<15$. Combining these requirements, the domain is the interval $(0, 5)$.

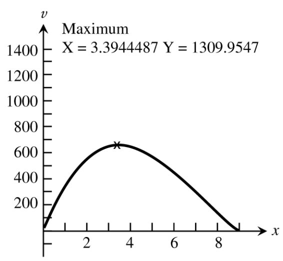


(c) The maximum volume is approximately 66.02 in.^{3} when $x \approx 1.96$ in.

(d) $V'(x) = 6x^2 - 50x + 75$. The critical point occurs when $V'(x) = 0$, at $x = \frac{50 \pm \sqrt{(-50)^2 - 4(6)(75)}}{2(6)} = \frac{50 \pm \sqrt{700}}{12}$
$= \frac{25 \pm 5\sqrt{7}}{6}$, that is, $x \approx 1.96$ or $x \approx 6.37$. We discard the larger value because it is not in the domain. Since $V''(x) = 12x - 50$, which is negative when $x \approx 1.96$, the critical point corresponds to the maximum volume. The maximum volume occurs when $x = \frac{25 - 5\sqrt{7}}{6} \approx 1.96$, which comfimrs the result in (c).

17. (a) The "sides" of the suitcase will measure $24 - 2x$ in. by $18 - 2x$ in. and will be $2x$ in. apart, so the volume formula is $V(x) = 2x(24 - 2x)(18 - 2x) = 8x^3 - 168x^2 + 862x$.

(b) We require $x > 0$, $2x < 18$, and $2x < 12$. Combining these requirements, the domain is the interval $(0, 9)$.



(c) The maximum volume is approximately 1309.95 in.^{3} when $x \approx 3.39$ in.

(d) $V'(x) = 24x^2 - 336x + 864 = 24(x^2 - 14x + 36)$. The critical point is at $x = \frac{14 \pm \sqrt{(-14)^2 - 4(1)(36)}}{2(1)} = \frac{14 \pm \sqrt{52}}{2}$
$= 7 \pm \sqrt{13}$, that is, $x \approx 3.39$ or $x \approx 10.61$. We discard the larger value because it is not in the domain. Since $V''(x) = 24(2x - 14)$ which is negative when $x \approx 3.39$, the critical point corresponds to the maximum volume. The maximum value occurs at $x = 7 - \sqrt{13} \approx 3.39$, which confirms the results in (c).

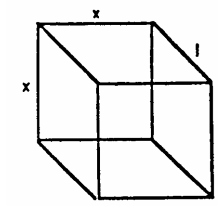
(e) $8x^3 - 168x^2 + 862x = 1120 \Rightarrow 8(x^3 - 21x^2 + 108x - 140) = 0 \Rightarrow 8(x - 2)(x - 5)(x - 14) = 0$. Since 14 is not in the fomain, the possible values of x are $x = 2$ in. or $x = 5$ in.

(f) The dimensions of the resulting box are $2x$ in., $(24 - 2x)$ in., and $(18 - 2x)$. Each of these measurements must be positive, so that gives the domain of $(0, 9)$.

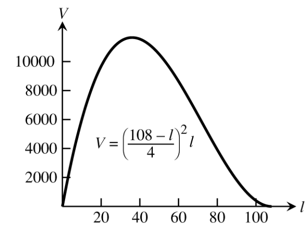
18. If the upper right vertex of the rectangle is located at $(x, 4\cos 0.5x)$ for $0 < x < \pi$, then the rectangle has width $2x$ and height $4\cos 0.5x$, so the area is $A(x) = 8x\cos 0.5x$. Solving $A'(x) = 0$ graphically for $0 < x < \pi$, we find that $x \approx 2.214$. Evaluating $2x$ and $4\cos 0.5x$ for $x \approx 2.214$, the dimensions of the rectangle are approximately 4.43 (width) by 1.79 (height), and the maximum area is approximately 7.923.

19. Let the radius of the cylinder be r cm, $0 < r < 10$. Then the height is $2\sqrt{100 - r^2}$ and the volume is $V(r) = 2\pi r^2\sqrt{100 - r^2}$ cm^3. Then, $V'(r) = 2\pi r^2\left(\frac{1}{\sqrt{100 - r^2}}\right)(-2r) + \left(2\pi\sqrt{100 - r^2}\right)(2r)$
$= \frac{-2\pi r^3 + 4\pi r(100 - r^2)}{\sqrt{100 - r^2}} = \frac{2\pi r(200 - 3r^2)}{\sqrt{100 - r^2}}$. The critical point for $0 < r < 10$ occurs at $r = \sqrt{\frac{200}{3}} = 10\sqrt{\frac{2}{3}}$. Since $V'(r) > 0$ for $0 < r < 10\sqrt{\frac{2}{3}}$ and $V'(r) < 0$ for $10\sqrt{\frac{2}{3}} < r < 10$, the critical point corresponds to the maximum volume. The dimensions are $r = 10\sqrt{\frac{2}{3}} \approx 8.16$ cm and $h = \frac{20}{\sqrt{3}} \approx 11.55$ cm, and the volume is $\frac{4000\pi}{3\sqrt{3}} \approx 2418.40$ cm^3.

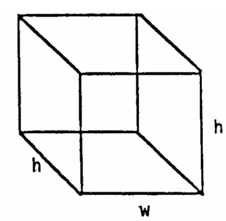
20. (a) From the diagram we have $4x + \ell = 108$ and $V = x^2\ell$. The volume of the box is $V(x) = x^2(108 - 4x)$, where $0 \le x < 27$. Then $V'(x) = 216x - 12x^2 = 12x(18 - x) = 0$ $\Rightarrow$ the critical points are 0 and 18, but $x = 0$ results in no box. Since $V''(x) = 216 - 24x < 0$ at $x = 18$ we have a maximum. The dimensions of the box are $18 \times 18 \times 36$ in.



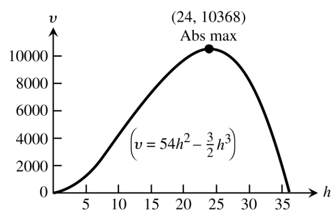
(b) In terms of length, $V(\ell) = x^2\ell = \left(\frac{108-\ell}{4}\right)^2 \ell$. The graph indicates that the maximum volume occurs near $\ell = 36$, which is consistent with the result of part (a).



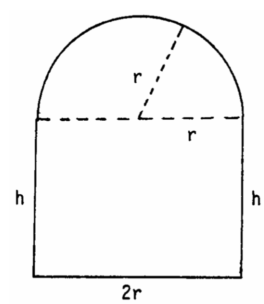
21. (a) From the diagram we have $3h + 2w = 108$ and $V = h^2w \Rightarrow V(h) = h^2\left(54 - \frac{3}{2}h\right) = 54h^2 - \frac{3}{2}h^3$. Then $V'(h) = 108h - \frac{9}{2}h^2 = \frac{9}{2}h(24 - h) = 0$ $\Rightarrow$ $h = 0$ or $h = 24$, but $h = 0$ results in no box. Since $V''(h) = 108 - 9h < 0$ at $h = 24$, we have a maximum volume at $h = 24$ and $w = 54 - \frac{3}{2}h = 18$.



(b)



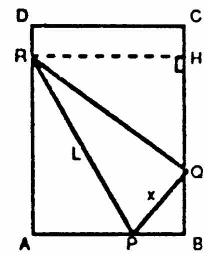
22. From the diagram the perimeter is $P = 2r + 2h + \pi r$, where r is the radius of the semicircle and h is the height of the rectangle. The amount of light transmitted proportional to
$A = 2rh + \frac{1}{4}\pi r^2 = r(P - 2r - \pi r) + \frac{1}{4}\pi r^2$
$= rP - 2r^2 - \frac{3}{4}\pi r^2$. Then $\frac{dA}{dr} = P - 4r - \frac{3}{2}\pi r = 0$
$\Rightarrow r = \frac{2P}{8+3\pi} \Rightarrow 2h = P - \frac{4P}{8+3\pi} - \frac{2\pi P}{8+3\pi} = \frac{(4+\pi)P}{8+3\pi}$.
Therefore, $\frac{2r}{h} = \frac{8}{4+\pi}$ gives the proportions that admit the <u>most</u> light since $\frac{d^2A}{dr^2} = -4 - \frac{3}{2}\pi < 0$.



23. The fixed volume is $V = \pi r^2 h + \frac{2}{3}\pi r^3 \Rightarrow h = \frac{V}{\pi r^2} - \frac{2r}{3}$, where h is the height of the cylinder and r is the radius of the hemisphere. To minimize the cost we must minimize surface area of the cylinder added to twice the surface area of the hemisphere. Thus, we minimize $C = 2\pi rh + 4\pi r^2 = 2\pi r\left(\frac{V}{\pi r^2} - \frac{2r}{3}\right) + 4\pi r^2 = \frac{2V}{r} + \frac{8}{3}\pi r^2$. Then $\frac{dC}{dr} = -\frac{2V}{r^2} + \frac{16}{3}\pi r = 0 \Rightarrow V = \frac{8}{3}\pi r^3 \Rightarrow r = \left(\frac{3V}{8\pi}\right)^{1/3}$. From the volume equation, $h = \frac{V}{\pi r^2} - \frac{2r}{3}$
$= \frac{4V^{1/3}}{\pi^{1/3} \cdot 3^{2/3}} - \frac{2 \cdot 3^{1/3} \cdot V^{1/3}}{3 \cdot 2 \cdot \pi^{1/3}} = \frac{3^{1/3} \cdot 2 \cdot 4 \cdot V^{1/3} - 2 \cdot 3^{1/3} \cdot V^{1/3}}{3 \cdot 2 \cdot \pi^{1/3}} = \left(\frac{3V}{\pi}\right)^{1/3}$. Since $\frac{d^2C}{dr^2} = \frac{4V}{r^3} + \frac{16}{3}\pi > 0$, these dimensions do minimize the cost.

24. The volume of the trough is maximized when the area of the cross section is maximized. From the diagram the area of the cross section is $A(\theta) = \cos\theta + \sin\theta\cos\theta$, $0 < \theta < \frac{\pi}{2}$. Then $A'(\theta) = -\sin\theta + \cos^2\theta - \sin^2\theta$ $= -(2\sin^2\theta + \sin\theta - 1) = -(2\sin\theta - 1)(\sin\theta + 1)$ so $A'(\theta) = 0 \Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = -1 \Rightarrow \theta = \frac{\pi}{6}$ because $\sin\theta \neq -1$ when $0 < \theta < \frac{\pi}{2}$. Also, $A'(\theta) > 0$ for $0 < \theta < \frac{\pi}{6}$ and $A'(\theta) < 0$ for $\frac{\pi}{6} < \theta < \frac{\pi}{2}$. Therefore, at $\theta = \frac{\pi}{6}$ there is a maximum.

25. (a) From the diagram we have: $\overline{AP} = x$, $\overline{RA} = \sqrt{L - x^2}$, $\overline{PB} = 8.5 - x$, $\overline{CH} = \overline{DR} = 11 - \overline{RA} = 11 - \sqrt{L - x^2}$, $\overline{QB} = \sqrt{x^2 - (8.5 - x)^2}$, $\overline{HQ} = 11 - \overline{CH} - \overline{QB}$ $= 11 - \left[11 - \sqrt{L - x^2} + \sqrt{x^2 - (8.5 - x)^2}\right]$ $= \sqrt{L - x^2} - \sqrt{x^2 - (8.5 - x)^2}$, $\overline{RQ}^2 = \overline{RH}^2 + \overline{HQ}^2$ $= (8.5)^2 + \left(\sqrt{L - x^2} - \sqrt{x^2 - (8.5 - x)^2}\right)^2$. It



follows that $\overline{RP}^2 = \overline{PQ}^2 + \overline{RQ}^2 \Rightarrow L^2 = x^2 + \left(\sqrt{L^2 - x^2} - \sqrt{x^2 - (x - 8.5)^2}\right)^2 + (8.5)^2$

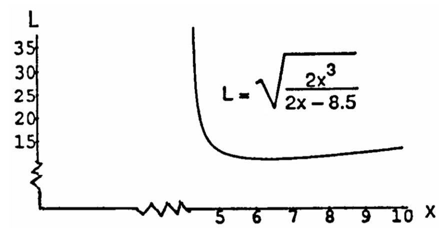
$\Rightarrow L^2 = x^2 + L^2 - x^2 - 2\sqrt{L^2 - x^2}\,\sqrt{17x - (8.5)^2} + 17x - (8.5)^2 + (8.5)^2$

$\Rightarrow 17^2x^2 = 4\left(L^2 - x^2\right)\left(17x - (8.5)^2\right) \Rightarrow L^2 = x^2 + \frac{17^2x^2}{4\left[17x - (8.5)^2\right]} = \frac{17x^3}{17x - (8.5)^2} = \frac{17x^3}{17x - \left(\frac{17}{2}\right)^2}$

$= \frac{4x^3}{4x - 17} = \frac{2x^3}{2x - 8.5}$.

(b) If $f(x) = \frac{4x^3}{4x - 17}$ is minimized, then L^2 is minimized. Now $f'(x) = \frac{4x^2(8x - 51)}{(4x - 17)^2} \Rightarrow f'(x) < 0$ when $x < \frac{51}{8}$ and $f'(x) > 0$ when $x > \frac{51}{8}$. Thus L^2 is minimized when $x = \frac{51}{8}$.

(c) When $x = \frac{51}{8}$, then $L \approx 11.0$ in.



26. (a) From the figure in the text we have $P = 2x + 2y \Rightarrow y = \frac{P}{2} - x$. If $P = 36$, then $y = 18 - x$. When the cylinder is formed, $x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$ and $h = y \Rightarrow h = 18 - x$. The volume of the cylinder is $V = \pi r^2 h$ $\Rightarrow V(x) = \frac{18x^2 - x^3}{4\pi}$. Solving $V'(x) = \frac{3x(12 - x)}{4\pi} = 0 \Rightarrow x = 0$ or 12; but when $x = 0$, there is no cylinder. Then $V''(x) = \frac{3}{\pi}\left(3 - \frac{x}{2}\right) \Rightarrow V''(12) < 0 \Rightarrow$ there is a maximum at $x = 12$. The values of $x = 12$ cm and $y = 6$ cm give the largest volume.

(b) In this case $V(x) = \pi x^2(18 - x)$. Solving $V'(x) = 3\pi x(12 - x) = 0 \Rightarrow x = 0$ or 12; but $x = 0$ would result in no cylinder. Then $V''(x) = 6\pi(6 - x) \Rightarrow V''(12) < 0 \Rightarrow$ there is a maximum at $x = 12$. The values of $x = 12$ cm and $y = 6$ cm give the largest volume.

27. Note that $h^2 + r^2 = 3$ and so $r = \sqrt{3 - h^2}$. Then the volume is given by $V = \frac{\pi}{3}r^2h = \frac{\pi}{3}(3 - h^2)h = \pi h - \frac{\pi}{3}h^3$ for $0 < h < \sqrt{3}$, and so $\frac{dV}{dh} = \pi - \pi r^2 = \pi(1 - r^2)$. The critical point (for $h > 0$) occurs at $h = 1$. Since $\frac{dV}{dh} > 0$ for $0 < h < 1$, and $\frac{dV}{dh} < 0$ for $1 < h < \sqrt{3}$, the critical point corresponds to the maximum volume. The cone of greatest volume has radius $\sqrt{2}$ m, height 1m, and volume $\frac{2\pi}{3}$ m^3.

28. Let $d = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$ and $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow y = -\frac{b}{a}x + b$. We can minimize d by minimizing $D = \left(\sqrt{x^2 + y^2}\right)^2 = x^2 + \left(-\frac{b}{a}x + b\right)^2 \Rightarrow D' = 2x + 2\left(-\frac{b}{a}x + b\right)\left(-\frac{b}{a}\right) = 2x + \frac{2b^2}{a^2}x - \frac{2b^2}{a}$. $D' = 0$ $\Rightarrow 2\left(x + \frac{b^2}{a^2}x - \frac{b^2}{a}\right) = 0 \Rightarrow x = \frac{ab^2}{a^2 + b^2}$ is the critical point $\Rightarrow y = -\frac{b}{a}\left(\frac{ab^2}{a^2 + b^2}\right) + b = \frac{a^2b}{a^2 + b^2}$.

$D'' = 2 + \frac{2b^2}{a^2} \Rightarrow D''\left(\frac{ab^2}{a^2+b^2}\right) = 2 + \frac{2b^2}{a^2} > 0 \Rightarrow$ the critical point is local minimum $\Rightarrow \left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$ is the point on the line $\frac{x}{a} + \frac{y}{b} = 1$ that is closest to the origin.

29. Let $S(x) = x + \frac{1}{x}, x > 0 \Rightarrow S'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$. $S'(x) = 0 \Rightarrow \frac{x^2-1}{x^2} = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$. Since $x > 0$, we only consider $x = 1$. $S''(x) = \frac{2}{x^3} \Rightarrow S''(1) = \frac{2}{1^3} > 0 \Rightarrow$ local minimum when $x = 1$

30. Let $S(x) = \frac{1}{x} + 4x^2, x > 0 \Rightarrow S'(x) = -\frac{1}{x^2} + 8x = \frac{8x^3-1}{x^2}$. $S'(x) = 0 \Rightarrow \frac{8x^3-1}{x^2} = 0 \Rightarrow 8x^3 - 1 = 0 \Rightarrow x = \frac{1}{2}$. $S''(x) = \frac{2}{x^3} + 8 \Rightarrow S''\left(\frac{1}{2}\right) = \frac{2}{(1/2)^3} + 8 > 0 \Rightarrow$ local minimum when $x = \frac{1}{2}$.

31. The length of the wire b = perimeter of the triangle + circumference of the circle. Let x = length of a side of the equilateral triangle $\Rightarrow P = 3x$, and let r = radius of the circle $\Rightarrow C = 2\pi r$. Thus $b = 3x + 2\pi r \Rightarrow r = \frac{b-3x}{2\pi}$. The area of the circle is πr^2 and the area of an equilateral triangle whose sides are x is $\frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$. Thus, the total area is given by $A = \frac{\sqrt{3}}{4}x^2 + \pi r^2 = \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{b-3x}{2\pi}\right)^2 = \frac{\sqrt{3}}{4}x^2 + \frac{(b-3x)^2}{4\pi} \Rightarrow A' = \frac{\sqrt{3}}{2}x - \frac{3}{2\pi}(b-3x) = \frac{\sqrt{3}}{2}x - \frac{3b}{2\pi} + \frac{9}{2\pi}x$
$A' = 0 \Rightarrow \frac{\sqrt{3}}{2}x - \frac{3b}{2\pi} + \frac{9}{2\pi}x = 0 \Rightarrow x = \frac{3b}{\sqrt{3}\pi+9}$. $A'' = \frac{\sqrt{3}}{2} + \frac{9}{2\pi} > 0 \Rightarrow$ local minimum at the critical point.
$P = 3\left(\frac{3b}{\sqrt{3}\pi+9}\right) = \frac{9b}{\sqrt{3}\pi+9}$ m is the length of the trianglular segment and $C = 2\pi\left(\frac{b-3x}{2\pi}\right) = b - 3x$
$= b - \frac{9b}{\sqrt{3}\pi+9} = \frac{\sqrt{3}\pi b}{\sqrt{3}\pi+9}$ m is the length of the circular segment.

32. The length of the wire b = perimeter of the square + circunference of the circle. Let x = length of a side of the square $\Rightarrow P = 4x$, and let r = radius of the circle $\Rightarrow C = 2\pi r$. Thus $b = 4x + 2\pi r \Rightarrow r = \frac{b-4x}{2\pi}$. The area of the circle is πr^2 and the area of a square whose sides are x is x^2. Thus, the total area is given by $A = x^2 + \pi r^2$
$= x^2 + \pi\left(\frac{b-4x}{2\pi}\right)^2 = x^2 + \frac{(b-4x)^2}{4\pi} \Rightarrow A' = 2x - \frac{4}{2\pi}(b-4x) = 2x - \frac{2b}{\pi} + \frac{8}{\pi}x$, $A' = 0 \Rightarrow 2x - \frac{2b}{\pi} + \frac{8}{\pi}x = 0$
$\Rightarrow x = \frac{b}{4+\pi}$. $A'' = 2 + \frac{8}{\pi} > 0 \Rightarrow$ local minimum at the critical point. $P = 4\left(\frac{b}{4+\pi}\right) = \frac{4b}{4+\pi}$ m is the length of the square segment and $C = 2\pi\left(\frac{b-4x}{2\pi}\right) = b - 4x = b - \frac{4b}{4+\pi} = \frac{b\pi}{4+\pi}$ m is the length of the circular segment.

33. Let $(x, y) = \left(x, \frac{4}{3}x\right)$ be the coordinates of the corner that intersects the line. Then base $= 3 - x$ and height $= y = \frac{4}{3}x$, thus the area of therectangle is given by $A = (3-x)\left(\frac{4}{3}x\right) = 4x - \frac{4}{3}x^2, 0 \le x \le 3$. $A' = 4 - \frac{8}{3}x$, $A' = 0 \Rightarrow x = \frac{3}{2}$. $A'' = -\frac{4}{3}$ $\Rightarrow A''\left(\frac{3}{2}\right) < 0 \Rightarrow$ local maximum at the critical point. The base $= 3 - \frac{3}{2} = \frac{3}{2}$ and the height $= \frac{4}{3}\left(\frac{3}{2}\right) = 2$.

34. Let $(x, y) = \left(x, \sqrt{9-x^2}\right)$ be the coordinates of the corner that intersects the semicircle. Then base $= 2x$ and height $= y$ $= \sqrt{9-x^2}$, thus the area of the inscribed rectangle is given by $A = (2x)\sqrt{9-x^2}, 0 \le x \le 3$. Then
$A' = 2\sqrt{9-x^2} + (2x)\frac{-x}{\sqrt{9-x^2}} = \frac{2(9-x^2)-2x^2}{\sqrt{9-x^2}} = \frac{18-4x^2}{\sqrt{4-x^2}}$, $A' = 0 \Rightarrow 18 - 4x^2 = 0 \Rightarrow x = \pm\frac{3\sqrt{2}}{2}$, only $x = \frac{3\sqrt{2}}{2}$ lies in $0 \le x \le 3$. A is continuous on the closed interval $0 \le x \le 3 \Rightarrow$ A has an absolute maxima and absolute minima.
$A(0) = 0$, $A(3) = 0$, and $A\left(\frac{3\sqrt{2}}{2}\right) = \left(3\sqrt{2}\right)\left(\frac{3\sqrt{2}}{2}\right) = 9 \Rightarrow$ absolute maxima. Base of rectangle is $3\sqrt{2}$ and height is $\frac{3\sqrt{2}}{2}$.

35. (a) $f(x) = x^2 + \frac{a}{x} \Rightarrow f'(x) = x^{-2}(2x^3 - a)$, so that $f'(x) = 0$ when $x = 2$ implies $a = 16$
(b) $f(x) = x^2 + \frac{a}{x} \Rightarrow f''(x) = 2x^{-3}(x^3 + a)$, so that $f''(x) = 0$ when $x = 1$ implies $a = -1$

36. If $f(x) = x^3 + ax^2 + bx$, then $f'(x) = 3x^2 + 2ax + b$ and $f''(x) = 6x + 2a$.

(a) A local maximum at $x = -1$ and local minimum at $x = 3 \Rightarrow f'(-1) = 0$ and $f'(3) = 0 \Rightarrow 3 - 2a + b = 0$ and $27 + 6a + b = 0 \Rightarrow a = -3$ and $b = -9$.

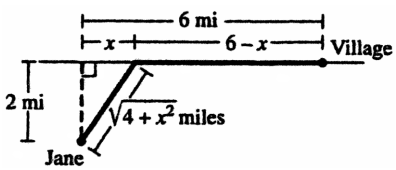
(b) A local minimum at $x = 4$ and a point of inflection at $x = 1 \Rightarrow f'(4) = 0$ and $f''(1) = 0 \Rightarrow 48 + 8a + b = 0$ and $6 + 2a = 0 \Rightarrow a = -3$ and $b = -24$.

37. (a) $s(t) = -16t^2 + 96t + 112 \Rightarrow v(t) = s'(t) = -32t + 96$. At $t = 0$, the velocity is $v(0) = 96$ ft/sec.

(b) The maximum height ocurs when $v(t) = 0$, when $t = 3$. The maximum height is $s(3) = 256$ ft and it occurs at $t = 3$ sec.

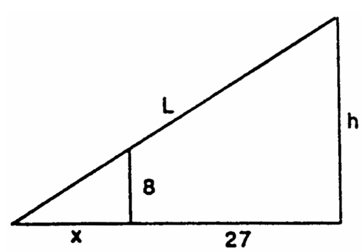
(c) Note that $s(t) = -16t^2 + 96t + 112 = -16(t+1)(t-7)$, so $s = 0$ at $t = -1$ or $t = 7$. Choosing the positive value of t, the velocity when $s = 0$ is $v(7) = -128$ ft/sec.

38.



Let x be the distance from the point on the shoreline nearest Jane's boat to the point where she lands her boat. Then she needs to row $\sqrt{4+x^2}$ mi at 2 mph and walk $6 - x$ mi at 5 mph. The total amount of time to reach the village is $f(x) = \frac{\sqrt{4+x^2}}{2} + \frac{6-x}{5}$ hours $(0 \le x \le 6)$. Then $f'(x) = \frac{1}{2}\frac{1}{2\sqrt{4+x^2}}(2x) - \frac{1}{5} = \frac{x}{2\sqrt{4+x^2}} - \frac{1}{5}$. Solving $f'(x) = 0$, we have: $\frac{x}{2\sqrt{4+x^2}} = \frac{1}{5} \Rightarrow 5x = 2\sqrt{4+x^2} \Rightarrow 25x^2 = 4(4+x^2) \Rightarrow 21x^2 = 16 \Rightarrow x = \pm\frac{4}{\sqrt{21}}$. We discard the negative value of x because it is not in the domain. Checking the endpoints and critical point, we have $f(0) = 2.2$, $f\left(\frac{4}{\sqrt{21}}\right) \approx 2.12$, and $f(6) \approx 3.16$. Jane should land her boat $\frac{4}{\sqrt{21}} \approx 0.87$ miles down the shoreline from the point nearest her boat.

39. $\frac{8}{x} = \frac{h}{x+27} \Rightarrow h = 8 + \frac{216}{x}$ and $L(x) = \sqrt{h^2 + (x+27)^2}$ $= \sqrt{\left(8 + \frac{216}{x}\right)^2 + (x+27)^2}$ when $x \ge 0$. Note that $L(x)$ is minimized when $f(x) = \left(8 + \frac{216}{x}\right)^2 + (x+27)^2$ is minimized. If $f'(x) = 0$, then $2\left(8 + \frac{216}{x}\right)\left(-\frac{216}{x^2}\right) + 2(x+27) = 0$ $\Rightarrow (x+27)\left(1 - \frac{1728}{x^3}\right) = 0 \Rightarrow x = -27$ (not acceptable since distance is never negative or $x = 12$. Then $L(12) = \sqrt{2197} \approx 46.87$ ft.



40. (a) $s_1 = s_2 \Rightarrow \sin t = \sin\left(t + \frac{\pi}{3}\right) \Rightarrow \sin t = \sin t \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos t \Rightarrow \sin t = \frac{1}{2}\sin t + \frac{\sqrt{3}}{2}\cos t \Rightarrow \tan t = \sqrt{3}$ $\Rightarrow t = \frac{\pi}{3}$ or $\frac{4\pi}{3}$

(b) The distance between the particles is $s(t) = |s_1 - s_2| = \left|\sin t - \sin\left(t + \frac{\pi}{3}\right)\right| = \frac{1}{2}\left|\sin t - \sqrt{3}\cos t\right|$

$\Rightarrow s'(t) = \frac{\left(\sin t - \sqrt{3}\cos t\right)\left(\cos t + \sqrt{3}\sin t\right)}{2\left|\sin t - \sqrt{3}\cos t\right|}$ since $\frac{d}{dx}|x| = \frac{x}{|x|} \Rightarrow$ critical times and endpoints are $0, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}, 2\pi$; then $s(0) = \frac{\sqrt{3}}{2}$, $s\left(\frac{\pi}{3}\right) = 0$, $s\left(\frac{5\pi}{6}\right) = 1$, $s\left(\frac{4\pi}{3}\right) = 0$, $s\left(\frac{11\pi}{6}\right) = 1$, $s(2\pi) = \frac{\sqrt{3}}{2} \Rightarrow$ the greatest distance between the particles is 1.

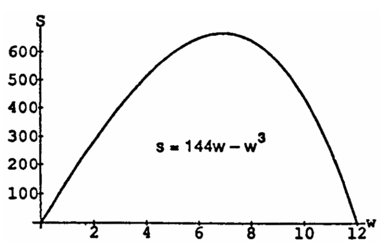
(c) Since $s'(t) = \frac{\left(\sin t - \sqrt{3}\cos t\right)\left(\cos t + \sqrt{3}\sin t\right)}{2\left|\sin t - \sqrt{3}\cos t\right|}$ we can conclude that at $t = \frac{\pi}{3}$ and $\frac{4\pi}{3}$, $s'(t)$ has cusps and the distance between the particles is changing the fastest near these points.

41. $I = \frac{k}{d^2}$, let x = distance the point is from the stronger light source $\Rightarrow 6 - x$ = distance the point is from the other light source. The intensity of illumination at the point from the stronger light is $I_1 = \frac{k_1}{x^2}$, and intensity of illumination at the point from the weaker light is $I_2 = \frac{k_2}{(6-x)^2}$. Since the intensity of the first light is eight times the intensity of the second light $\Rightarrow k_1 = 8k_2$. $\Rightarrow I_1 = \frac{8k_2}{x^2}$. The total intensity is given by $I = I_1 + I_2 = \frac{8k_2}{x^2} + \frac{k_2}{(6-x)^2} \Rightarrow I' = -\frac{16k_2}{x^3} + \frac{2k_2}{(6-x)^3}$ $= \frac{-16(6-x)^3 k_2 + 2x^3 k_2}{x^3 (6-x)^3}$ and $I' = 0 \Rightarrow \frac{-16(6-x)^3 k_2 + 2x^3 k_2}{x^3 (6-x)^3} = 0 \Rightarrow -16(6-x)^3 k_2 + 2x^3 k_2 = 0 \Rightarrow x = 4$ m. $I'' = \frac{48k_2}{x^4} + \frac{6k_2}{(6-x)^4}$ $\Rightarrow I''(4) = \frac{48k_2}{4^4} + \frac{6k_2}{(6-4)^4} > 0 \Rightarrow$ local minimum. The point should be 4 m from the stronger light source.

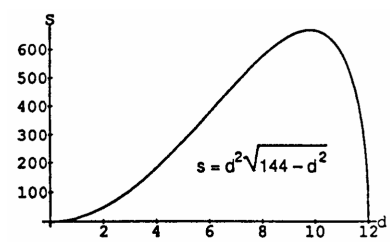
42. $R = \frac{v_0^2}{g}\sin 2\alpha \Rightarrow \frac{dR}{d\alpha} = \frac{2v_0^2}{g}\cos 2\alpha$ and $\frac{dR}{d\alpha} = 0 \Rightarrow \frac{2v_0^2}{g}\cos 2\alpha = 0 \Rightarrow \alpha = \frac{\pi}{4}$. $\frac{d^2R}{d\alpha^2} = -\frac{4v_0^2}{g}\sin 2\alpha \Rightarrow \left.\frac{d^2R}{d\alpha^2}\right|_{\alpha=\frac{\pi}{4}} = -\frac{4v_0^2}{g}\sin 2\left(\frac{\pi}{4}\right)$ $= -\frac{4v_0^2}{g} < 0 \Rightarrow$ local maximum. Thus, the firing angle of $\alpha = \frac{\pi}{4} = 45°$ will maximize the range R.

43. (a) From the diagram we have $d^2 = 4r^2 - w^2$. The strength of the beam is $S = kwd^2 = kw\left(4r^2 - w^2\right)$. When $r = 6$, then $S = 144kw - kw^3$. Also, $S'(w) = 144k - 3kw^2 = 3k\left(48 - w^2\right)$ so $S'(w) = 0 \Rightarrow w = \pm 4\sqrt{3}$; $S''\left(4\sqrt{3}\right) < 0$ and $-4\sqrt{3}$ is not acceptable. Therefore $S\left(4\sqrt{3}\right)$ is the maximum strength. The dimensions of the strongest beam are $4\sqrt{3}$ by $4\sqrt{6}$ inches.

(b)



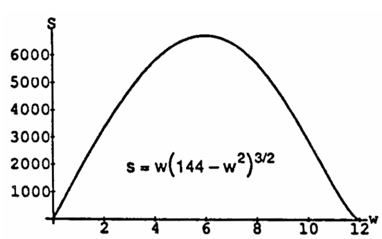
(c)



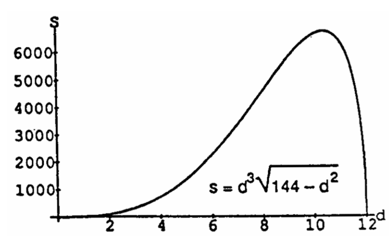
Both graphs indicate the same maximum value and are consistent with each other. Changing k does not change the dimensions that give the strongest beam (i.e., do not change the values of w and d that produce the strongest beam).

44. (a) From the situation we have $w^2 = 144 - d^2$. The stiffness of the beam is $S = kwd^3 = kd^3\left(144 - d^2\right)^{1/2}$, where $0 \le d \le 12$. Also, $S'(d) = \frac{4kd^2\left(108 - d^2\right)}{\sqrt{144 - d^2}} \Rightarrow$ critical points at 0, 12, and $6\sqrt{3}$. Both $d = 0$ and $d = 12$ cause $S = 0$. The maximum occurs at $d = 6\sqrt{3}$. The dimensions are 6 by $6\sqrt{3}$ inches.

(b)



(c)



Both graphs indicate the same maximum value and are consistent with each other. The changing of k has no effect.

45. (a) $s = 10\cos(\pi t) \Rightarrow v = -10\pi\sin(\pi t) \Rightarrow$ speed $= |10\pi\sin(\pi t)| = 10\pi|\sin(\pi t)| \Rightarrow$ the maximum speed is $10\pi \approx 31.42$ cm/sec since the maximum value of $|\sin(\pi t)|$ is 1; the cart is moving the fastest at t = 0.5 sec, 1.5 sec, 2.5 sec and 3.5 sec when $|\sin(\pi t)|$ is 1. At these times the distance is $s = 10\cos\left(\frac{\pi}{2}\right) = 0$ cm and $a = -10\pi^2\cos(\pi t) \Rightarrow |a| = 10\pi^2|\cos(\pi t)| \Rightarrow |a| = 0$ cm/sec^2

(b) $|a| = 10\pi^2|\cos(\pi t)|$ is greatest at t = 0.0 sec, 1.0 sec, 2.0 sec, 3.0 sec and 4.0 sec, and at these times the magnitude of the cart's position is $|s| = 10$ cm from the rest position and the speed is 0 cm/sec.

46. (a) $2 \sin t = \sin 2t \Rightarrow 2 \sin t - 2 \sin t \cos t = 0 \Rightarrow (2 \sin t)(1 - \cos t) = 0 \Rightarrow t = k\pi$ where k is a positive integer

(b) The vertical distance between the masses is $s(t) = |s_1 - s_2| = \left((s_1 - s_2)^2\right)^{1/2} = \left((\sin 2t - 2 \sin t)^2\right)^{1/2}$

$\Rightarrow\ s'(t) = \left(\frac{1}{2}\right)\left((\sin 2t - 2 \sin t)^2\right)^{-1/2}(2)(\sin 2t - 2 \sin t)(2 \cos 2t - 2 \cos\ t) = \frac{2(\cos 2t - \cos t)(\sin 2t - 2 \sin t)}{|\sin 2t - 2 \sin t|}$

$= \frac{4(2 \cos t + 1)(\cos t - 1)(\sin t)(\cos t - 1)}{|\sin 2t - 2 \sin t|} \Rightarrow$ critical times at $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$; then $s(0) = 0$,

$s\left(\frac{2\pi}{3}\right) = \left|\sin\left(\frac{4\pi}{3}\right) - 2 \sin\left(\frac{2\pi}{3}\right)\right| = \frac{3\sqrt{3}}{2}$, $s(\pi) = 0$, $s\left(\frac{4\pi}{3}\right) = \left|\sin\left(\frac{8\pi}{3}\right) - 2 \sin\left(\frac{4\pi}{3}\right)\right| = \frac{3\sqrt{3}}{2}$, $s(2\pi) = 0$

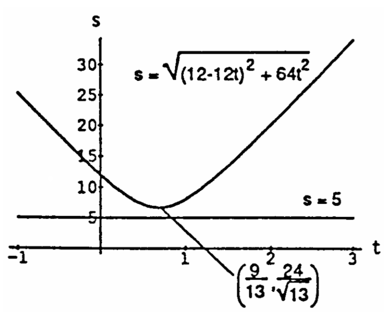
$\Rightarrow$ the greatest distance is $\frac{3\sqrt{3}}{2}$ at $t = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

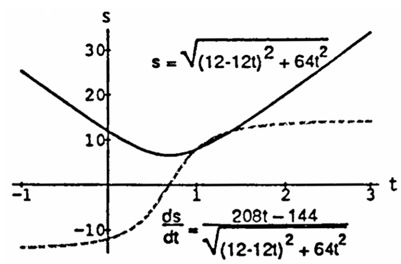
47. (a) $s = \sqrt{(12 - 12t)^2 + (8t)^2} = \left((12 - 12t)^2 + 64t^2\right)^{1/2}$

(b) $\frac{ds}{dt} = \frac{1}{2}\left((12 - 12t)^2 + 64t^2\right)^{-1/2}[2(12 - 12t)(-12) + 128t] = \frac{208t - 144}{\sqrt{(12 - 12t)^2 + 64t^2}} \Rightarrow \left.\frac{ds}{dt}\right|_{t=0} = -12$ knots and $\left.\frac{ds}{dt}\right|_{t=1} = 8$ knots

(c) The graph indicates that the ships did not see each other because $s(t) > 5$ for all values of t.

(d) The graph supports the conclusions in parts (b) and (c).

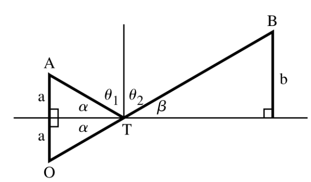




(e) $\lim\limits_{t \to \infty} \frac{ds}{dt} = \sqrt{\lim\limits_{t \to \infty} \frac{(208t - 144)^2}{144(1 - t)^2 + 64t^2}} = \sqrt{\lim\limits_{t \to \infty} \frac{\left(208 - \frac{144}{t}\right)^2}{144\left(\frac{1}{t} - 1\right)^2 + 64}} = \sqrt{\frac{208^2}{144 + 64}} = \sqrt{208} = 4\sqrt{13}$

which equals the square root of the sums of the squares of the individual speeds.

48. The distance $\overline{OT} + \overline{TB}$ is minimized when $\overline{OB}$ is a straight line. Hence $\angle\alpha = \angle\beta \Rightarrow \theta_1 = \theta_2$.



49. If $v = kax - kx^2$, then $v' = ka - 2kx$ and $v'' = -2k$, so $v' = 0 \Rightarrow x = \frac{a}{2}$. At $x = \frac{a}{2}$ there is a maximum since $v''\left(\frac{a}{2}\right) = -2k < 0$. The maximum value of v is $\frac{ka^2}{4}$.

50. (a) According to the graph, $y'(0) = 0$.

(b) According to the graph, $y'(-L) = 0$.

(c) $y(0) = 0$, so $d = 0$. Now $y'(x) = 3ax^2 + 2bx + c$, so $y'(0) = 0$ implies that $c = 0$. There fore, $y(x) = ax^3 + bx^2$ and $y'(x) = 3ax^2 + 2bx$. Then $y(-L) = -aL^3 + bL^2 = H$ and $y'(-L) = 3aL^2 - 2bL = 0$, so we have two linear equations in two unknowns a and b. The second equation gives $b = \frac{3aL}{2}$. Substituting into the first equation, we have $-aL^3 + \frac{3aL^3}{2} = H$, or $\frac{aL^3}{2} = H$, so $a = 2\frac{H}{L^3}$. Therefore, $b = 3\frac{H}{L^2}$ and the equation for y is $y(x) = 2\frac{H}{L^3}x^3 + 3\frac{H}{L^2}x^2$, or $y(x) = H\left[2\left(\frac{x}{L}\right)^3 + 3\left(\frac{x}{L}\right)^2\right]$.

51. The profit is $p = nx - nc = n(x-c) = \left[a(x-c)^{-1} + b(100-x)\right](x-c) = a + b(100-x)(x-c)$ $= a + (bc + 100b)x - 100bc - bx^2$. Then $p'(x) = bc + 100b - 2bx$ and $p''(x) = -2b$. Solving $p'(x) = 0 \Rightarrow x = \frac{c}{2} + 50$. At $x = \frac{c}{2} + 50$ there is a maximum profit since $p''(x) = -2b < 0$ for all x.

52. Let x represent the number of people over 50. The profit is $p(x) = (50 + x)(200 - 2x) - 32(50 + x) - 6000$ $= -2x^2 + 68x + 2400$. Then $p'(x) = -4x + 68$ and $p'' = -4$. Solving $p'(x) = 0 \Rightarrow x = 17$. At $x = 17$ there is a maximum since $p''(17) < 0$. It would take 67 people to maximize the profit.

53. (a) $A(q) = kmq^{-1} + cm + \frac{h}{2}q$, where $q > 0 \Rightarrow A'(q) = -kmq^{-2} + \frac{h}{2} = \frac{hq^2 - 2km}{2q^2}$ and $A''(q) = 2kmq^{-3}$. The critical points are $-\sqrt{\frac{2km}{h}}$, 0, and $\sqrt{\frac{2km}{h}}$, but only $\sqrt{\frac{2km}{h}}$ is in the domain. Then $A''\left(\sqrt{\frac{2km}{h}}\right) > 0 \Rightarrow$ at $q = \sqrt{\frac{2km}{h}}$ there is a minimum average weekly cost.

(b) $A(q) = \frac{(k+bq)m}{q} + cm + \frac{h}{2}q = kmq^{-1} + bm + cm + \frac{h}{2}q$, where $q > 0 \Rightarrow A'(q) = 0$ at $q = \sqrt{\frac{2km}{h}}$ as in (a). Also $A''(q) = 2kmq^{-3} > 0$ so the most economical quantity to order is still $q = \sqrt{\frac{2km}{h}}$ which minimizes the average weekly cost.

54. We start with $c(x) =$ the cost of producing x items, $x > 0$, and $\frac{c(x)}{x} =$ the average cost of producing x items, assumed to be differentiable. If the average cost can be minimized, it will be at a production level at which $\frac{d}{dx}\left(\frac{c(x)}{x}\right) = 0$ $\Rightarrow \frac{x\,c'(x) - c(x)}{x^2} = 0$ (by the quotient rule) $\Rightarrow x\,c'(x) - c(x) = 0$ (multiply both sides by x^2) $\Rightarrow c'(x) = \frac{c(x)}{x}$ where $c'(x)$ is the marginal cost. This concludes the proof. (Note: The theorem does not assure a production level that will give a minimum cost, but rather, it indicates where to look to see if there is one. Find the production levels where the average cost equals the marginal cost, then check to see if any of them give a mimimum.)

55. The profit $p(x) = r(x) - c(x) = 6x - (x^3 - 6x^2 + 15x) = -x^3 + 6x^2 - 9x$, where $x \ge 0$. Then $p'(x) = -3x^2 + 12x - 9$ $= -3(x-3)(x-1)$ and $p''(x) = -6x + 12$. The critical points are 1 and 3. Thus $p''(1) = 6 > 0 \Rightarrow$ at $x = 1$ there is a local minimum, and $p''(3) = -6 < 0 \Rightarrow$ at $x = 3$ there is a local maximum. But $p(3) = 0 \Rightarrow$ the best you can do is break even.

56. The average cost of producing x items is $\bar{c}(x) = \frac{c(x)}{x} = x^2 - 20x + 20{,}000 \Rightarrow \bar{c}'(x) = 2x - 20 = 0 \Rightarrow x = 10$, the only critical value. The average cost is $\bar{c}(10) = \$19{,}900$ per item is a minimum cost because $\bar{c}''(10) = 2 > 0$.

57. Let $x =$ the length of a side of the square base of the box and $h =$ the height of the box. $V = x^2h = 48 \Rightarrow h = \frac{48}{x^2}$. The total cost is given by $C = 6 \cdot x^2 + 4(4 \cdot x\,h) = 6x^2 + 16x\left(\frac{48}{x^2}\right) = 6x^2 + \frac{768}{x}$, $x > 0 \Rightarrow C' = 12x - \frac{768}{x^2} = \frac{12x^3 - 768}{x^2}$
$C' = 0 \Rightarrow \frac{12x^3 - 768}{x^2} = 0 \Rightarrow 12x^3 - 768 = 0 \Rightarrow x = 4$; $C'' = 12 + \frac{1536}{x^2} \Rightarrow C''(4) = 12 + \frac{1536}{4^2} > 0 \Rightarrow$ local minimum.
$x = 4 \Rightarrow h = \frac{48}{4^2} = 3$ and $C(4) = 6(4)^2 + \frac{768}{4} = 288 \Rightarrow$ the box is 4 ft $\times$ 4 ft $\times$ 3 ft, with a minimum cost of \$288

58. Let $x =$ the number of \$10 increases in the charge per room, then price per room $= 50 + 10x$, and the number of rooms filled each night $= 800 - 40x \Rightarrow$ the total revenue is $R(x) = (50 + 10x)(800 - 40x) = -400x^2 + 6000x + 40000$, $0 \le x \le 20 \Rightarrow R'(x) = -800x + 6000$; $R'(x) = 0 \Rightarrow -800x + 6000 = 0 \Rightarrow x = \frac{15}{2}$; $R''(x) = -800$ $\Rightarrow R''\left(\frac{15}{2}\right) = -800 < 0 \Rightarrow$ local maximum. The price per room is $50 + 10\left(\frac{15}{2}\right) = \$125$.

59. We have $\frac{dR}{dM} = CM - M^2$. Solving $\frac{d^2R}{dM^2} = C - 2M = 0 \Rightarrow M = \frac{C}{2}$. Also, $\frac{d^3R}{dM^3} = -2 < 0 \Rightarrow$ at $M = \frac{C}{2}$ there is a maximum.