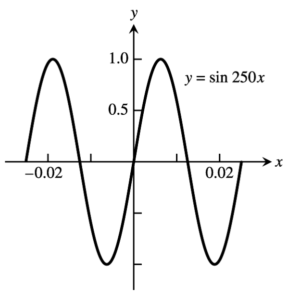
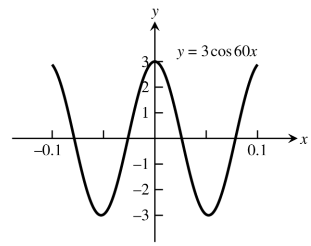
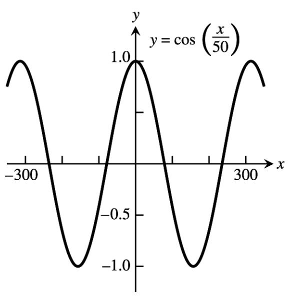
25. $[-0.03,\ 0.03]$ by $[-1.25,\ 1.25]$



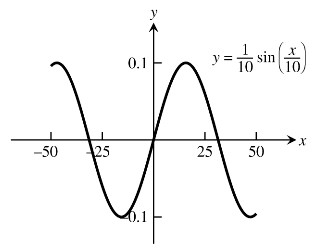
26. $[-0.1,\ 0.1]$ by $[-3,\ 3]$



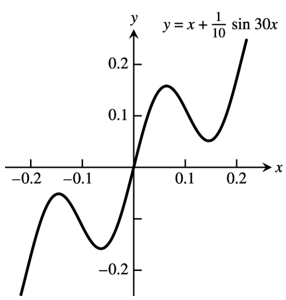
27. $[-300,\ 300]$ by $[-1.25,\ 1.25]$



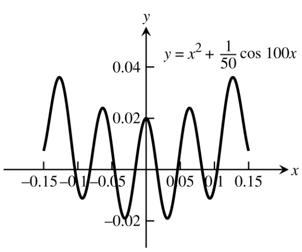
28. $[-50,\ 50]$ by $[-0.1,\ 0.1]$



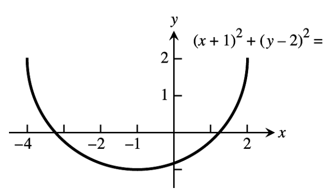
29. $[-0.25,\ 0.25]$ by $[-0.3,\ 0.3]$



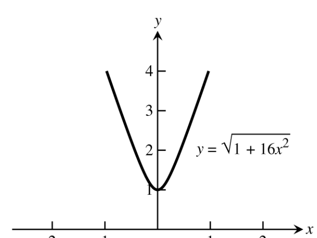
30. $[-0.15,\ 0.15]$ by $[-0.02,\ 0.05]$



31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$.
The lower half is produced by graphing
$y = 2 - \sqrt{-x^2 - 2x + 8}$.



32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm\sqrt{1 + 16x^2}$. The upper branch is produced by graphing $y = \sqrt{1 + 16x^2}$.



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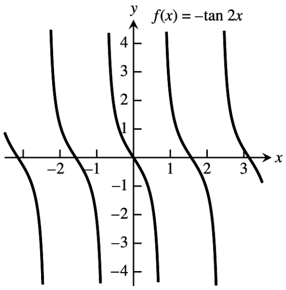

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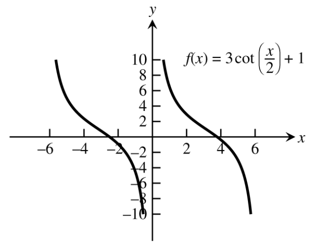

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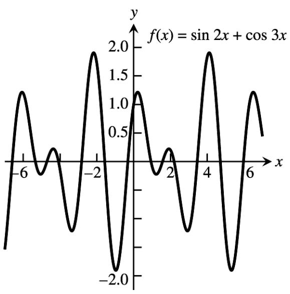

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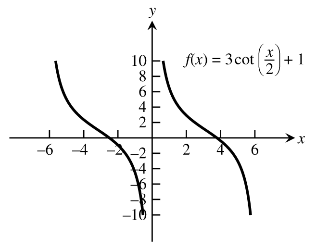

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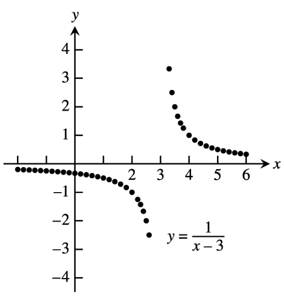

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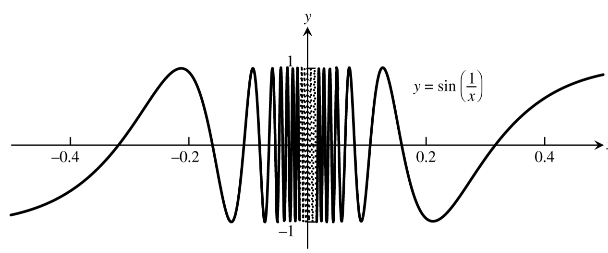

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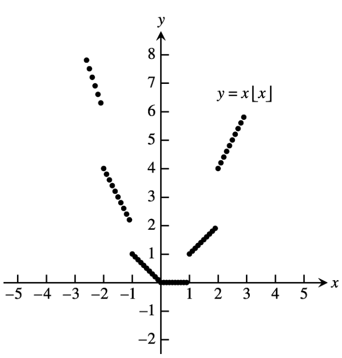

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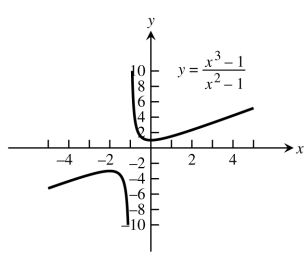


CHAPTER 1 PRACTICE EXERCISES

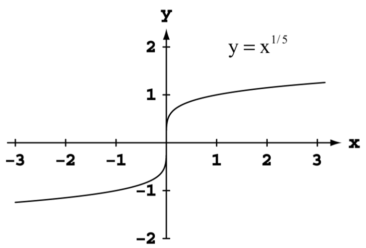
1. The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$.

2. The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$.

3. The coordinates of a point on the parabola are (x, x^2). The angle of inclination θ joining this point to the origin satisfies the equation $\tan\theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan\theta, \tan^2\theta)$.

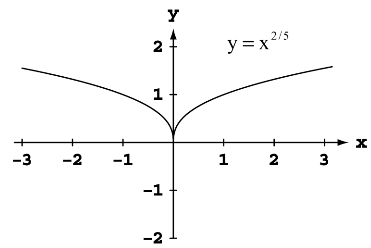
4. $\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500\tan\theta$ ft.

5.



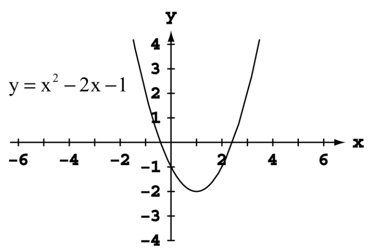
Symmetric about the origin.

6.



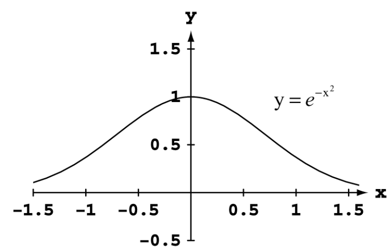
Symmetric about the y-axis.

7.



Neither

8.



Symmetric about the y-axis.

9. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$. Even.

10. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$. Odd.

11. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$. Even.

12. $y(-x) = \sec(-x)\tan(-x) = \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x} = -\sec x \tan x = -y(x)$. Odd.

13. $y(-x) = \frac{(-x)^4+1}{(-x)^3-2(-x)} = \frac{x^4+1}{-x^3+2x} = -\frac{x^4+1}{x^3-2x} = -y(x)$. Odd.

14. $y(-x) = (-x) - \sin(-x) = (-x) + \sin x = -(x - \sin x) = -y(x)$. Odd.

15. $y(-x) = -x + \cos(-x) = -x + \cos x$. Neither even nor odd.

16. $y(-x) = (-x)\cos(-x) = -x\cos x = -y(x)$. Odd.

17. Since f and g are odd $\Rightarrow f(-x) = -f(x)$ and $g(-x) = -g(x)$.
  (a) $(f \cdot g)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (f \cdot g)(x) \Rightarrow f \cdot g$ is even
  (b) $f^3(-x) = f(-x)f(-x)f(-x) = [-f(x)][-f(x)][-f(x)] = -f(x) \cdot f(x) \cdot f(x) = -f^3(x) \Rightarrow f^3$ is odd.
  (c) $f(\sin(-x)) = f(-\sin(x)) = -f(\sin(x)) \Rightarrow f(\sin(x))$ is odd.
  (d) $g(\sec(-x)) = g(\sec(x)) \Rightarrow g(\sec(x))$ is even.
  (e) $|g(-x)| = |-g(x)| = |g(x)| \Rightarrow |g|$ is even.

18. Let $f(a-x) = f(a+x)$ and define $g(x) = f(x+a)$. Then $g(-x) = f((-x)+a) = f(a-x) = f(a+x) = f(x+a) = g(x)$ $\Rightarrow g(x) = f(x+a)$ is even.

19. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

20. (a) Since the square root requires $1-x \geq 0$, the domain is $(-\infty, 1]$.
    (b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

21. (a) Since the square root requires $16-x^2 \geq 0$, the domain is $[-4, 4]$.
    (b) For values of x in the domain, $0 \leq 16-x^2 \leq 16$, so $0 \leq \sqrt{16-x^2} \leq 4$. The range is $[0, 4]$.

22. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

23. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

24. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.
    (b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

25. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The sine function attains values from -1 to 1, so $-2 \leq 2\sin(3x+\pi) \leq 2$ and hence $-3 \leq 2\sin(3x+\pi) - 1 \leq 1$. The range is $[-3, 1]$.

26. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

27. (a) The logarithm requires $x-3 > 0$, so the domain is $(3, \infty)$.
    (b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

28. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.
    (b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

29. (a) Increasing because volume increases as radius increases
    (b) Neither, since the greatest integer function is composed of horizontal (constant) line segments
    (c) Decreasing because as the height increases, the atmospheric pressure decreases.
    (d) Increasing because the kinetic (motion) energy increases as the particles velocity increases.

30. (a) Increasing on $[2, \infty)$
    (b) Increasing on $[-1, \infty)$
    (c) Increasing on $(-\infty, \infty)$
    (d) Increasing on $[\frac{1}{2}, \infty)$

31. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.
    (b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

32. (a) The function is defined for $-2 \le x \le 2$, so the domain is $[-2, 2]$.
    (b) The range is $[-1, 1]$.

33. First piece: Line through $(0, 1)$ and $(1, 0)$. $m = \frac{0-1}{1-0} = \frac{-1}{1} = -1 \Rightarrow y = -x + 1 = 1 - x$
    Second piece: Line through $(1, 1)$ and $(2, 0)$. $m = \frac{0-1}{2-1} = \frac{-1}{1} = -1 \Rightarrow y = -(x-1) + 1 = -x + 2 = 2 - x$
    $f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$

34. First piece: Line through $(0, 0)$ and $(2, 5)$. $m = \frac{5-0}{2-0} = \frac{5}{2} \Rightarrow y = \frac{5}{2}x$
    Second piece: Line through $(2, 5)$ and $(4, 0)$. $m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}(x-2) + 5 = -\frac{5}{2}x + 10 = 10 - \frac{5x}{2}$
    $f(x) = \begin{cases} \frac{5}{2}x, & 0 \le x < 2 \\ 10 - \frac{5x}{2}, & 2 \le x \le 4 \end{cases}$ (Note: $x = 2$ can be included on either piece.)

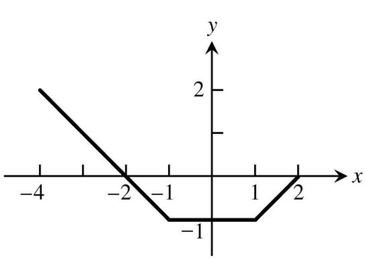
35. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$
    (b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{1}{2}+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$
    (c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \ne 0$
    (d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x+2}} + 2}} = \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$

36. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\sqrt[3]{-1+1}\right) = f(0) = 2 - 0 = 2$
    (b) $(g \circ f)(2) = f(g(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$
    (c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$
    (d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{x+1}\right) = \sqrt[3]{\sqrt[3]{x+1} + 1}$

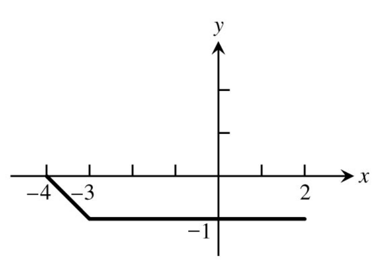
37. (a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x+2}\right) = 2 - \left(\sqrt{x+2}\right)^2 = -x,\ x \ge -2.$
    $(g \circ f)(x) = f(g(x)) = g(2 - x^2) = \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$
    (b) Domain of $f \circ g$: $[-2, \infty)$.
    Domain of $g \circ f$: $[-2, 2]$.
    (c) Range of $f \circ g$: $(-\infty, 2]$.
    Range of $g \circ f$: $[0, 2]$.

38. (a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{1-x}\right) = \sqrt{\sqrt{1-x}} = \sqrt[4]{1-x}.$
    $(g \circ f)(x) = f(g(x)) = g\left(\sqrt{x}\right) = \sqrt{1 - \sqrt{x}}$
    (b) Domain of $f \circ g$: $(-\infty, 1]$.
    Domain of $g \circ f$: $[0, 1]$.
    (c) Range of $f \circ g$: $[0, \infty)$.
    Range of $g \circ f$: $[0, 1]$.

39. $y = f(x)$



$y = (f \circ f)(x)$



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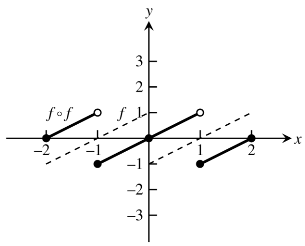

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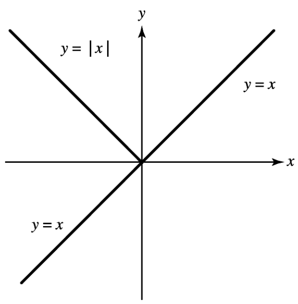
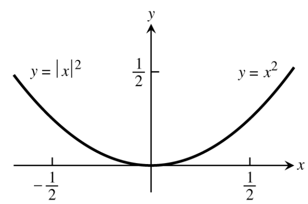


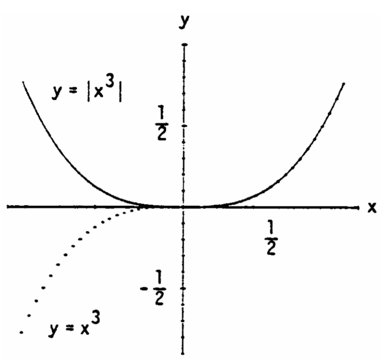
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

42.



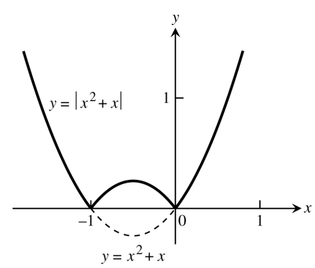
It does not change the graph.

43.



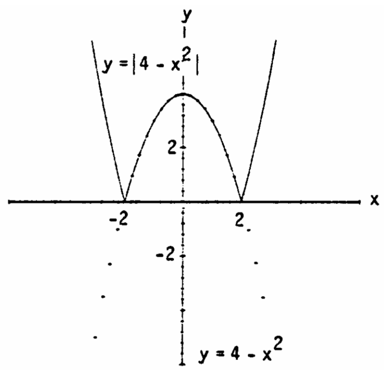
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

44.



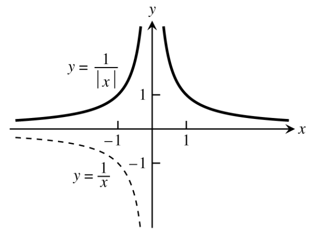
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

45.



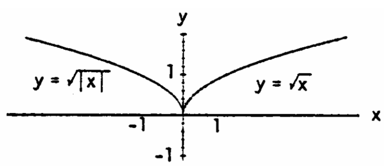
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

46.



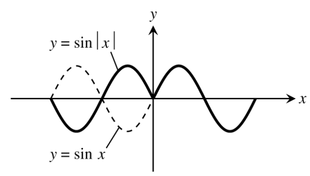
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

47.



The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

48.

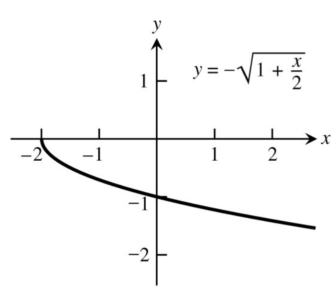


The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

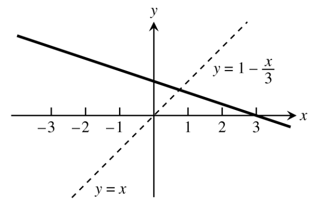
49. (a) $y = g(x-3) + \frac{1}{2}$
(b) $y = g\left(x + \frac{2}{3}\right) - 2$
(c) $y = g(-x)$
(d) $y = -g(x)$
(e) $y = 5 \cdot g(x)$
(f) $y = g(5x)$

50. (a) Shift the graph of f right 5 units
(b) Horizontally compress the graph of f by a factor of 4
(c) Horizontally compress the graph of f by a factor of 3 and a then reflect the graph about the y-axis
(d) Horizontally compress the graph of f by a factor of 2 and then shift the graph left $\frac{1}{2}$ unit.
(e) Horizontally stretch the graph of f by a factor of 3 and then shift the graph down 4 units.
(f) Vertically stretch the graph of f by a factor of 3, then reflect the graph about the x-axis, and finally shift the graph up $\frac{1}{4}$ unit.

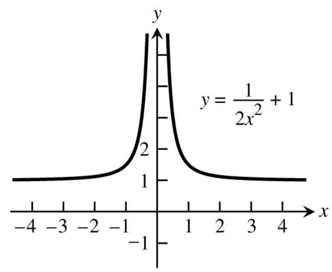
51. Reflection of the grpah of $y = \sqrt{x}$ about the x-axis followed by a horizontal compression by a factor of $\frac{1}{2}$ then a shift left 2 units.



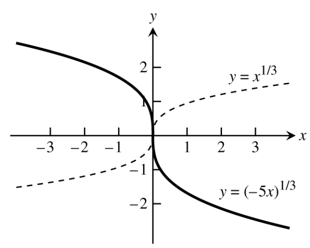
52. Reflect the graph of $y = x$ about the x-axis, followed by a vertical compression of the graph by a factor of 3, then shift the graph up 1 unit.



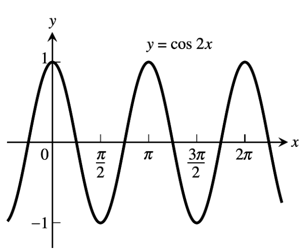
53. Vertical compression of the graph of $y = \frac{1}{x^2}$ by a factor of 2, then shift the graph up 1 unit.



54. Reflect the graph of $y = x^{1/3}$ about the y-axis, then compress the graph horizontally by a factor of 5.

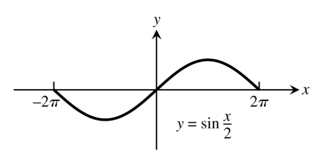


55.



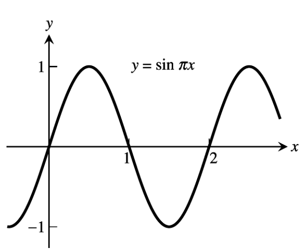
period $= \pi$

56.



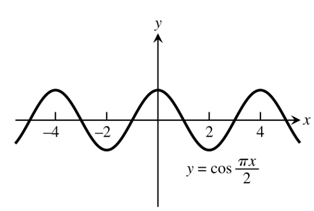
period $= 4\pi$

57.



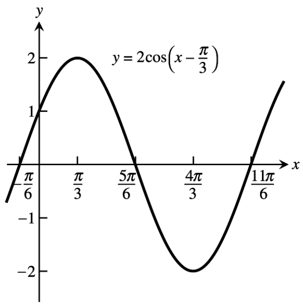
period $= 2$

58.



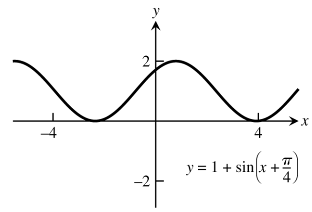
period $= 4$

59.



period $= 2\pi$

60.



period $= 2\pi$

61. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$. By the theorem of Pythagoras, $a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4-3} = 1$.

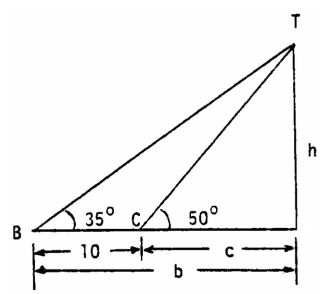
(b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

62. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$ (b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

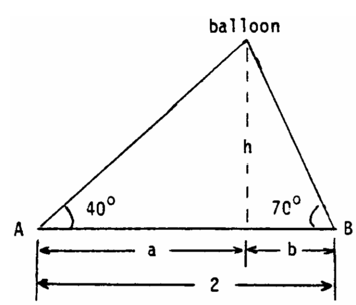
63. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$ (b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

64. (a) $\sin A = \frac{a}{c}$ (c) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

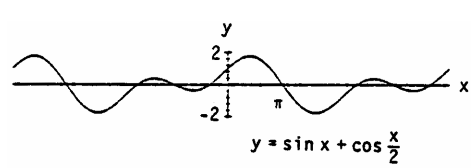
65. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flatground, respectively. Then, $\tan 50° = \frac{h}{c}$, $\tan 35° = \frac{h}{b}$, and $b - c = 10$.
Thus, $h = c \tan 50°$ and $h = b \tan 35° = (c + 10) \tan 35°$
$\Rightarrow c \tan 50° = (c + 10) \tan 35°$
$\Rightarrow c (\tan 50° - \tan 35°) = 10 \tan 35°$
$\Rightarrow c = \frac{10 \tan 35°}{\tan 50° - \tan 35°} \Rightarrow h = c \tan 50°$
$= \frac{10 \tan 35° \tan 50°}{\tan 50° - \tan 35°} \approx 16.98$ m.



66. Let h = height of balloon above ground. From the figure at the right, $\tan 40° = \frac{h}{a}$, $\tan 70° = \frac{h}{b}$, and $a + b = 2$. Thus, $h = b \tan 70° \Rightarrow h = (2 - a) \tan 70°$ and $h = a \tan 40°$
$\Rightarrow (2 - a) \tan 70° = a \tan 40° \Rightarrow a(\tan 40° + \tan 70°)$
$= 2 \tan 70° \Rightarrow a = \frac{2 \tan 70°}{\tan 40° + \tan 70°} \Rightarrow h = a \tan 40°$
$= \frac{2 \tan 70° \tan 40°}{\tan 40° + \tan 70°} \approx 1.3$ km.



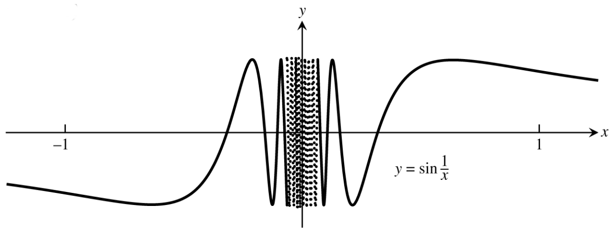
67. (a)



(b) The period appears to be 4π.

(c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x+4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos\frac{x}{2}$ since the period of sine and cosine is 2π. Thus, $f(x)$ has period 4π.

68. (a)



(b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$

(c) f is not periodic. For suppose f has period p. Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k. Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/2\pi)+kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/2\pi)+kp}\right) > 0$ which is a contradiction. Thus f has no period, as claimed.

CHAPTER 1 ADDITIONAL AND ADVANCED EXERCISES

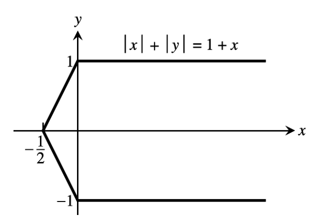
1. There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.

2. Yes, there are many such function pairs. For example, if $g(x) = (2x + 3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f\left((2x + 3)^3\right) = \left((2x + 3)^3\right)^{1/3} = 2x + 3$.

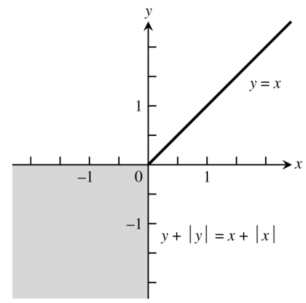
3. If f is odd and defined at x, then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2 \Rightarrow 4 = 0$, which is a contradiction. Also, $g(x)$ is not even unless $f(x) = 0$ for all x. On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

4. If g is odd and $g(0)$ is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

5. For (x, y) in the 1st quadrant, $|x| + |y| = 1 + x \Leftrightarrow x + y = 1 + x \Leftrightarrow y = 1$. For (x, y) in the 2nd quadrant, $|x| + |y| = x + 1 \Leftrightarrow -x + y = x + 1 \Leftrightarrow y = 2x + 1$. In the 3rd quadrant, $|x| + |y| = x + 1 \Leftrightarrow -x - y = x + 1 \Leftrightarrow y = -2x - 1$. In the 4th quadrant, $|x| + |y| = x + 1 \Leftrightarrow x + (-y) = x + 1 \Leftrightarrow y = -1$. The graph is given at the right.

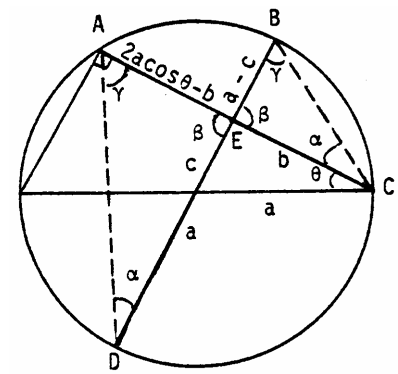


6. We use reasoning similar to Exercise 5.
   (1) 1st quadrant: $y + |y| = x + |x|$
   $\Leftrightarrow\ 2y = 2x\ \Leftrightarrow\ y = x.$
   (2) 2nd quadrant: $y + |y| = x + |x|$
   $\Leftrightarrow\ 2y = x + (-x) = 0\ \Leftrightarrow\ y = 0.$
   (3) 3rd quadrant: $y + |y| = x + |x|$
   $\Leftrightarrow\ y + (-y) = x + (-x)\ \Leftrightarrow\ 0 = 0$
   $\Rightarrow$ all points in the 3rd quadrant satisfy the equation.
   (4) 4th quadrant: $y + |y| = x + |x|$
   $\Leftrightarrow\ y + (-y) = 2x\ \Leftrightarrow\ 0 = x.$ Combining these results we have the graph given at the right:



7. (a) $\sin^2 x + \cos^2 x = 1\ \Rightarrow\ \sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x)\ \Rightarrow\ (1 - \cos x) = \frac{\sin^2 x}{1+\cos x}$
   $\Rightarrow\ \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$
   (b) Using the definition of the tangent function and the double angle formulas, we have
   $\tan^2\left(\frac{x}{2}\right) = \frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{\frac{1-\cos\left(2\left(\frac{x}{2}\right)\right)}{2}}{\frac{1+\cos\left(2\left(\frac{x}{2}\right)\right)}{2}} = \frac{1-\cos x}{1+\cos x}.$

8. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD. Similarly, the two angles labeled α are equal since they both subtend arc AB. Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b} = \frac{2a\cos\theta - b}{a+c}$
   $\Rightarrow\ (a - c)(a + c) = b(2a\cos\theta - b)$
   $\Rightarrow\ a^2 - c^2 = 2ab\cos\theta - b^2$
   $\Rightarrow\ c^2 = a^2 + b^2 - 2ab\cos\theta.$



9. As in the proof of the law of sines of Section 1.3, Exercise 61, $ah = bc\sin A = ab\sin C = ac\sin B$
   $\Rightarrow$ the area of ABC $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B.$

10. As in Section 1.3, Exercise 61, $(\text{Area of ABC})^2 = \frac{1}{4}(\text{base})^2(\text{height})^2 = \frac{1}{4}a^2h^2 = \frac{1}{4}a^2b^2\sin^2 C$
   $= \frac{1}{4}a^2b^2(1 - \cos^2 C)$. By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C\ \Rightarrow\ \cos C = \frac{a^2+b^2-c^2}{2ab}.$
   Thus, $(\text{area of ABC})^2 = \frac{1}{4}a^2b^2(1 - \cos^2 C) = \frac{1}{4}a^2b^2\left(1 - \left(\frac{a^2+b^2-c^2}{2ab}\right)^2\right) = \frac{a^2b^2}{4}\left(1 - \frac{(a^2+b^2-c^2)^2}{4a^2b^2}\right)$
   $= \frac{1}{16}\left(4a^2b^2 - (a^2+b^2-c^2)^2\right) = \frac{1}{16}\left[(2ab + (a^2+b^2-c^2))(2ab - (a^2+b^2-c^2))\right]$
   $= \frac{1}{16}\left[((a+b)^2 - c^2)(c^2 - (a-b)^2)\right] = \frac{1}{16}\left[((a+b)+c)((a+b)-c)(c+(a-b))(c-(a-b))\right]$
   $= \left[\left(\frac{a+b+c}{2}\right)\left(\frac{-a+b+c}{2}\right)\left(\frac{a-b+c}{2}\right)\left(\frac{a+b-c}{2}\right)\right] = s(s-a)(s-b)(s-c)$, where $s = \frac{a+b+c}{2}.$
   Therefore, the area of ABC equals $\sqrt{s(s-a)(s-b)(s-c)}$.

11. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x)\ \Rightarrow\ f(x) = -f(x)$ for all x in the domain of f. Thus $2f(x) = 0\ \Rightarrow\ f(x) = 0.$

12. (a) As suggested, let $E(x) = \frac{f(x)+f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x)+f(-(-x))}{2} = \frac{f(x)+f(-x)}{2} = E(x) \Rightarrow$ E is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x)+f(-x)}{2} = \frac{f(x)-f(-x)}{2}$. Then $O(-x) = \frac{f(-x)-f(-(-x))}{2} = \frac{f(-x)-f(x)}{2} = -\left(\frac{f(x)-f(-x)}{2}\right) = -O(x) \Rightarrow$ O is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.

(b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0 = (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x) = -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and $O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 11. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.

13. $y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

(a) If $a > 0$ the graph is a parabola that opens upward. Increasing a causes a vertical stretching and a shift of the vertex toward the y-axis and upward. If $a < 0$ the graph is a parabola that opens downward. Decreasing a causes a vertical stretching and a shift of the vertex toward the y-axis and downward.

(b) If $a > 0$ the graph is a parabola that opens upward. If also $b > 0$, then increasing b causes a shift of the graph downward to the left; if $b < 0$, then decreasing b causes a shift of the graph downward and to the right.

If $a < 0$ the graph is a parabola that opens downward. If $b > 0$, increasing b shifts the graph upward to the right. If $b < 0$, decreasing b shifts the graph upward to the left.

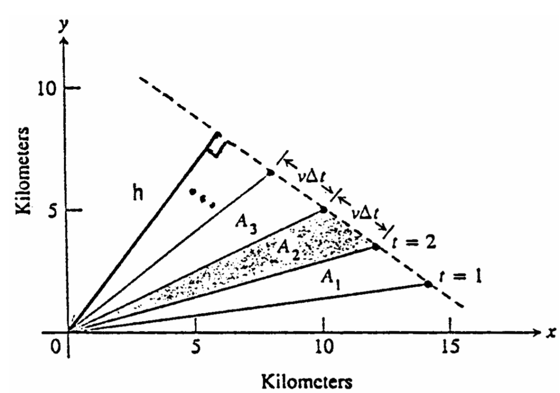
(c) Changing c (for fixed a and b) by Δc shifts the graph upward Δc units if $\Delta c > 0$, and downward $-\Delta c$ units if $\Delta c < 0$.

14. (a) If $a > 0$, the graph rises to the right of the vertical line $x = -b$ and falls to the left. If $a < 0$, the graph falls to the right of the line $x = -b$ and rises to the left. If $a = 0$, the graph reduces to the horizontal line $y = c$. As $|a|$ increases, the slope at any given point $x = x_0$ increases in magnitude and the graph becomes steeper. As $|a|$ decreases, the slope at x_0 decreases in magnitude and the graph rises or falls more gradually.

(b) Increasing b shifts the graph to the left; decreasing b shifts it to the right.

(c) Increasing c shifts the graph upward; decreasing c shifts it downward.

15. Each of the triangles pictured has the same base $b = v\Delta t = v(1 \text{ sec})$. Moreover, the height of each triangle is the same value h. Thus $\frac{1}{2}$(base)(height) $= \frac{1}{2}bh = A_1 = A_2 = A_3 = \ldots$. In conclusion, the object sweeps out equal areas in each one second interval.



16. (a) Using the midpoint formula, the coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of $\overline{OP} = \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.

(b) The slope of $\overline{AB} = \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments $\overline{AB}$ and $\overline{OP}$ are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, $\overline{AB}$ is perpendicular to $\overline{OP}$ when $a = b$.

17. From the figure we see that $0 \le \theta \le \frac{\pi}{2}$ and $AB = AD = 1$. From trigonometry we have the following: $\sin\theta = \frac{EB}{AB} = EB$, $\cos\theta = \frac{AE}{AB} = AE$, $\tan\theta = \frac{CD}{AD} = CD$, and $\tan\theta = \frac{EB}{AE} = \frac{\sin\theta}{\cos\theta}$. We can see that:
area $\triangle AEB <$ area sector $\widehat{DB} <$ area $\triangle ADC \Rightarrow \frac{1}{2}(AE)(EB) < \frac{1}{2}(AD)^2\theta < \frac{1}{2}(AD)(CD)$
$\Rightarrow \frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}(1)^2\theta < \frac{1}{2}(1)(\tan\theta) \Rightarrow \frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}\theta < \frac{1}{2}\frac{\sin\theta}{\cos\theta}$

18. $(f\circ g)(x) = f(g(x)) = a(cx+d)+b = acx+ad+b$ and $(g\circ f)(x) = g(f(x)) = c(ax+b)+d = acx+cb+d$
Thus $(f\circ g)(x) = (g\circ f)(x) \Rightarrow acx+ad+b = acx+bc+d \Rightarrow ad+b = bc+d$. Note that $f(d) = ad+b$ and $g(b) = cb+d$, thus $(f\circ g)(x) = (g\circ f)(x)$ if $f(d) = g(b)$.

NOTES: